The linear algebra of convection

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with

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Outline

- Linearity (!) of deep convection (anomalies)
- The system in matrix form: **M**
  - built in z or p basis, using CRM or SCM
  - eigenvector/eigenvalue basis

- Can we estimate **M** from observations?
  - (and model-M from GCM output in similar ways)?
    - → an important check, and a unique role for a GCM:
      - *Does GCM-output-diagnosed model-M agree with SCM-mapped model-M ?*

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Linearity (!) of convection

Sensitivities of Cumulus-Ensemble Rainfall in a Cloud-Resolving Model with Parameterized Large-Scale Dynamics

BRIAN E. MAPES

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My first grant! (NSF) Work done in mid- 1990s

(Manuscript received 1 August 2002, in final form 20 April 2004)

ABSTRACT

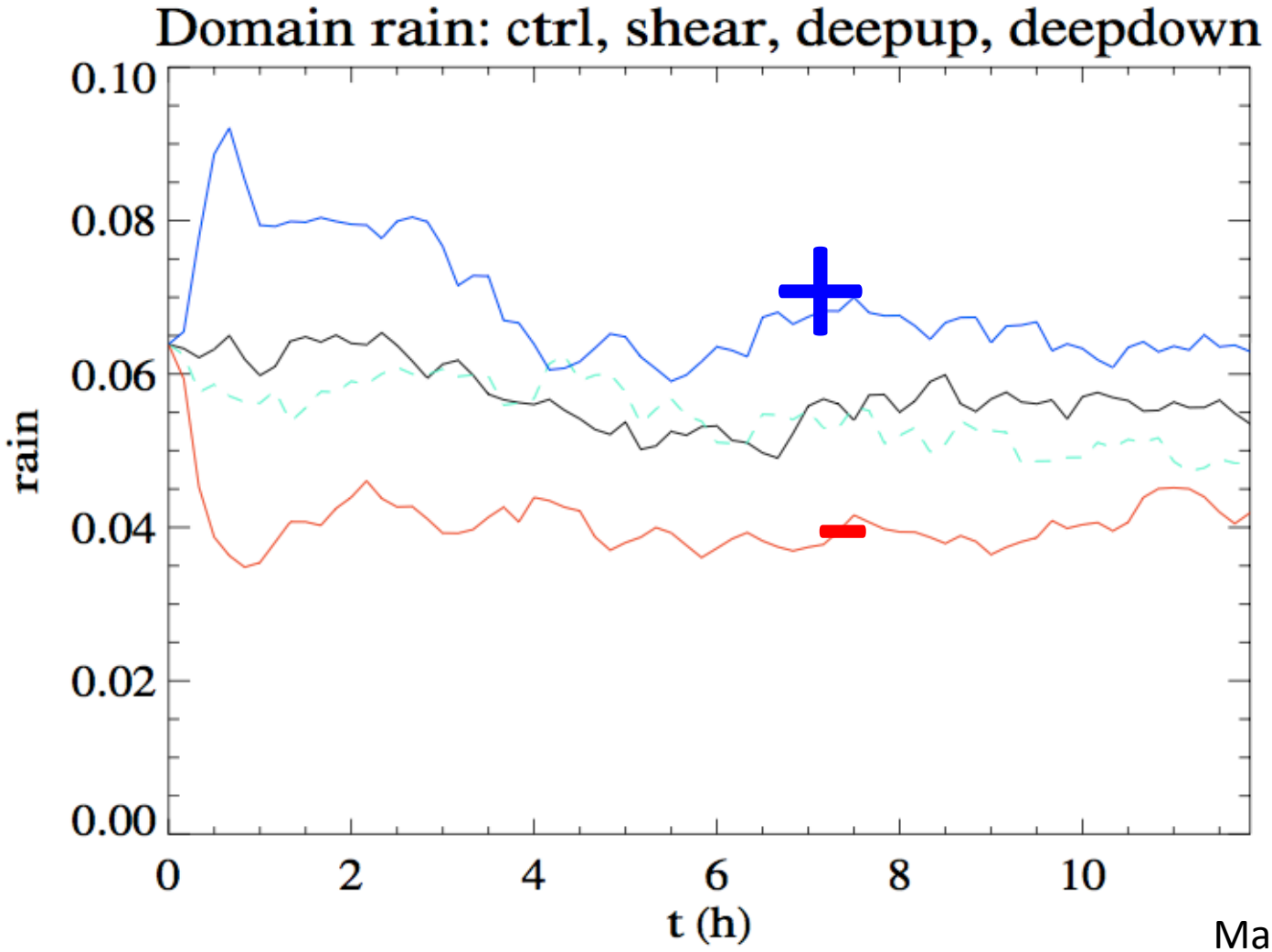
The problem of closure in cumulus parameterization requires an understanding of the sensitivities of convective cloud systems to their large-scale setting. As a step toward such an understanding, this study probes some sensitivities of a simulated ensemble of convective clouds in a two-dimensional cloud-resolving model (CRM). The ensemble is initially in statistical equilibrium with a steady imposed background forcing (cooling and moistening). Large-scale stimuli are imposed as horizontally uniform perturbations nudged into the model fields over 10 min, and the rainfall response of the model clouds is monitored.

Hypothesis:

low-level (Inhibition) control, not deep (CAPE) control

(Salvage writeup before moving to Miami. Not one of my better papers...)

first hint of linearity: response to equal and opposite forcings



Mapes 2004

A much better job of it

Transient Environmental Sensitivities of Explicitly Simulated Tropical Convection

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CIRES, University of Colorado, Boulder, Colorado

BRIAN E. MAPES

University of Miami, RSMAS, Miami, Florida

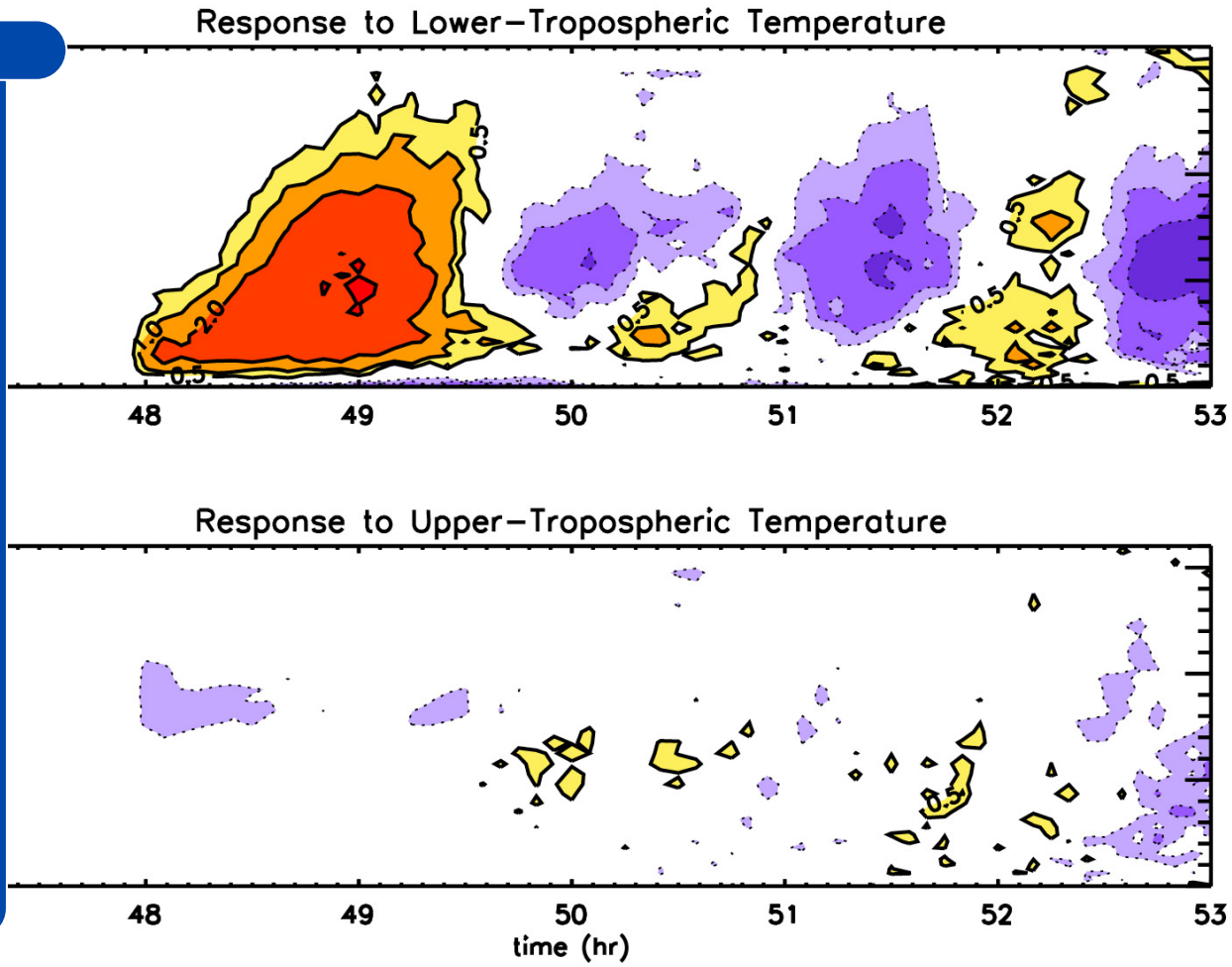
(Manuscript received 29 July 2009, in final form 13 October 2009)

ABSTRACT

A three-dimensional cloud-resolving model, maintained in a statistically steady convecting state by tropics-like forcing, is subjected to sudden (10 min) stimuli consisting of horizontally homogeneous temperature and/or moisture sources with various profiles. Ensembles of simulations are used to increase the statistical robustness of the results and to assess the deterministic nature of the model response for domain sizes near

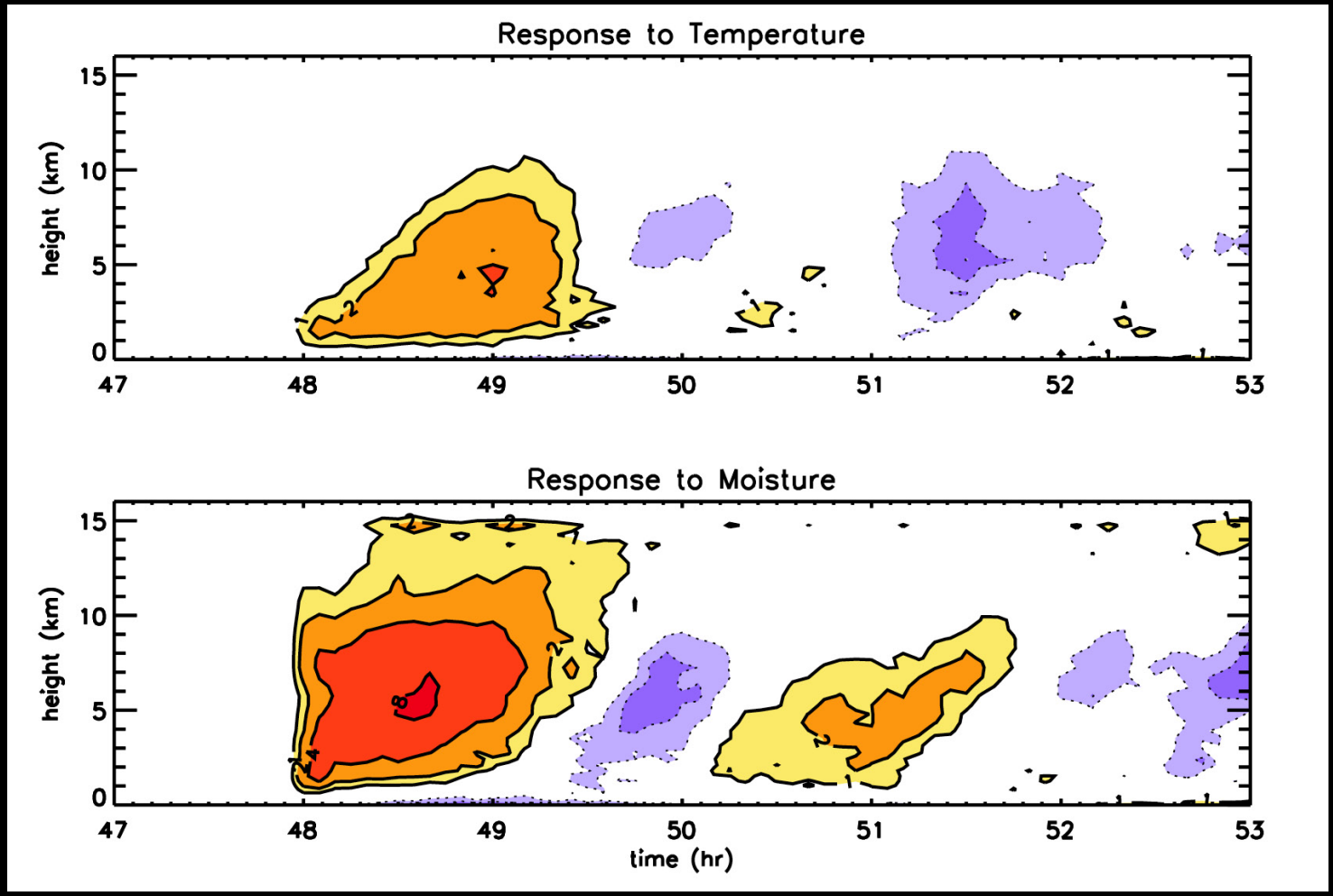
Heating rate responses to lower- and upper-tropospheric parts of imposed T'

low-level control hypothesis supported (on these scales...) (...where numerical cu param operates)



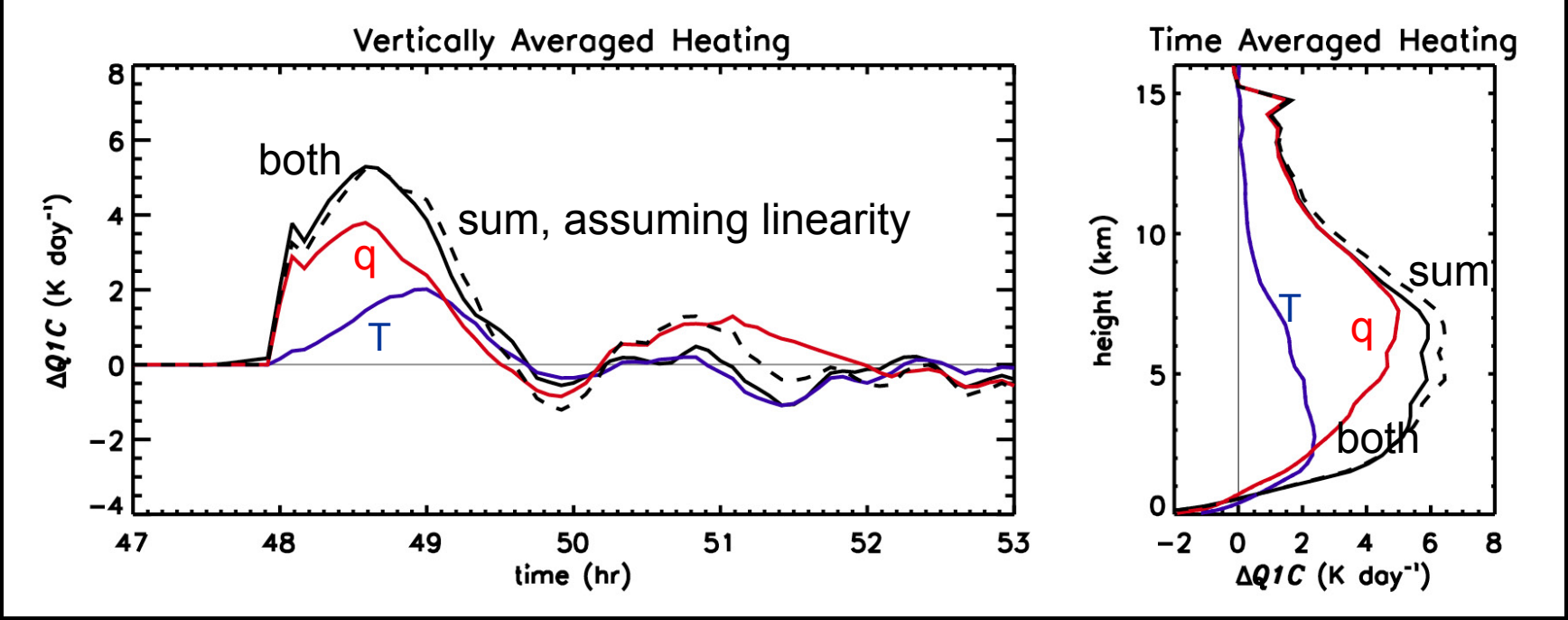
Courtesy Stefan Tulich (2006 AGU)

Temperature and Moisture Perturbations Imposed Separately



Courtesy Stefan Tulich (2006 AGU)

Linearity test:
$Q_1'(T',q') = Q_1'(T',0) + Q_1'(0,q')$?

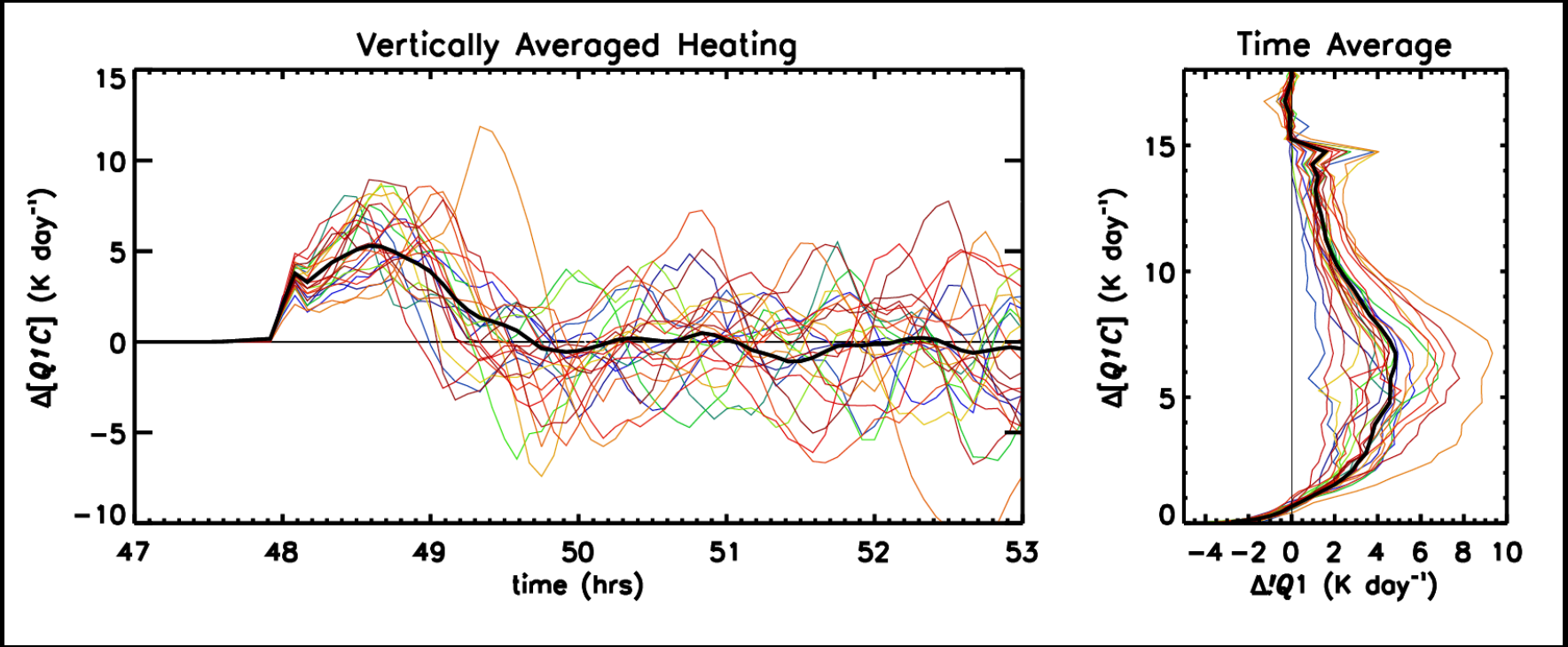


Yes: solid black curve resembles dotted curve in both time series & profile

Courtesy Stefan Tulich (2006 AGU)

Expectation value is linear, even when not purely deterministic

Ensemble Spread:

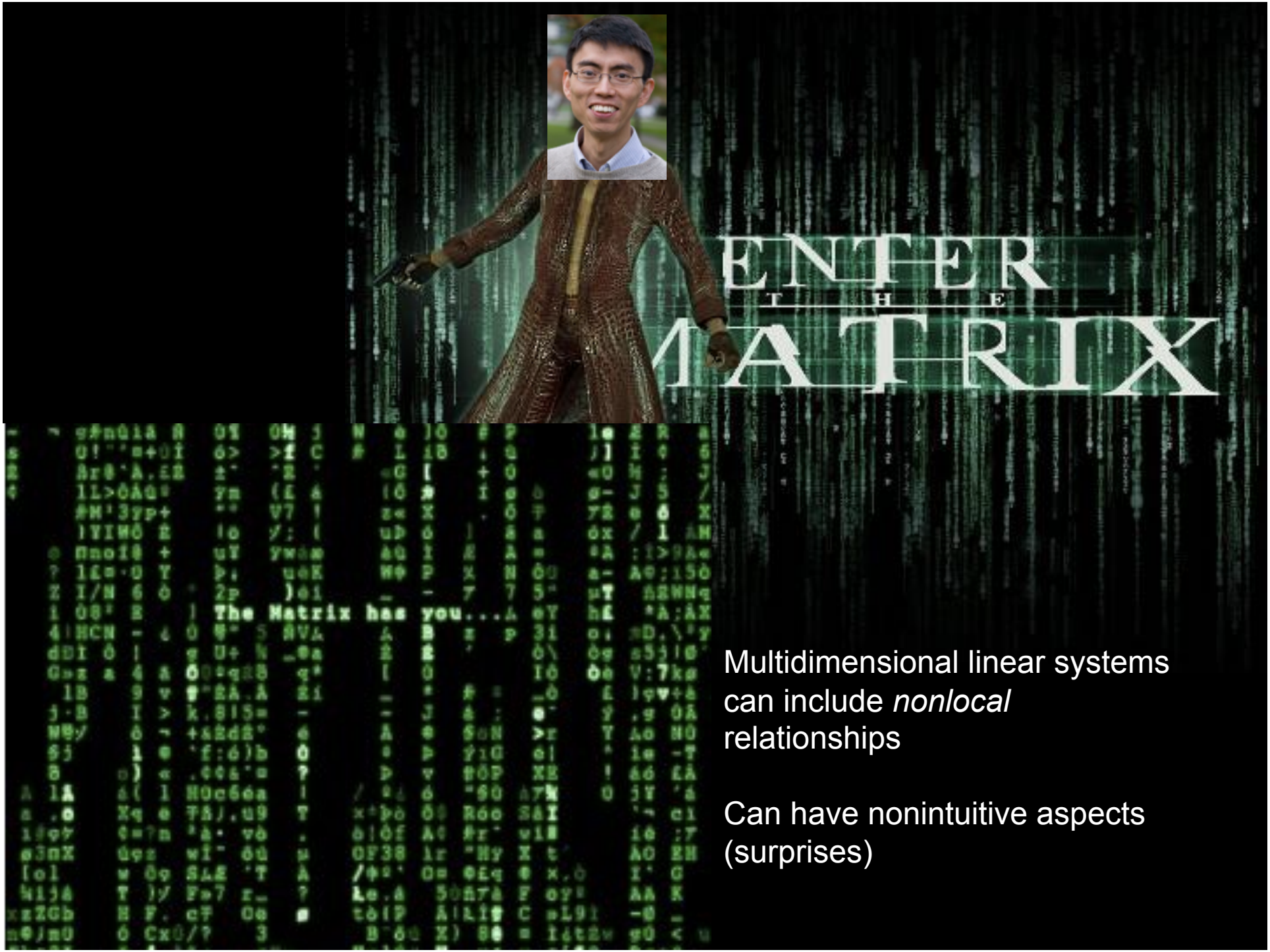


Courtesy Stefan Tulich (2006 AGU)

Outline

- Linearity (!) of deep convection (anomalies)
- The system in matrix form
  - in a spatial basis, using CRM & inversion (ZK)
    - could be done with SCM the same way...
  - math checks, eigenvector/eigenvalue basis, etc.

- Can we estimate M from obs?
  - (and model-M from GCM output in similar ways)?



Multidimensional linear systems can include *nonlocal* relationships

Can have nonintuitive aspects (surprises)

Kuang 2010 JAS

2. Method for constructing the linear response functions

Our goal is to derive a matrix **M** so that, given the anomalous state vector **x**, we can compute the anomalous convective tendencies as

$$\frac{d\mathbf{x}}{dt} = \mathbf{M}\mathbf{x}. \tag{1}$$

The state vector is considered here to include profiles of domain-averaged temperature T and specific humidity q anomalies (or their projections onto a set of basis func-

M is a *tangent linear convection parameterization*

$$\begin{bmatrix} \dot{T}_c'(p) \\ \dot{q}_c'(p) \end{bmatrix} = [M] \begin{bmatrix} T'(p) \\ q'(p) \end{bmatrix}$$

- Linearized about a steadily convecting base state
  - 2 tried in Kuang 2010
    - 1. RCE and 2. deep ascent-like forcing

Tulich and Mapes 2010 was a frontal assault on sensitivity

monitor convection response

$$\begin{bmatrix} \dot{T}_c'(p) \\ \dot{q}_c'(p) \end{bmatrix} = CRM\left(\begin{bmatrix} T'(p) \\ q'(p) \end{bmatrix} \right)$$

Inject stimuli suddenly

- Response is *time dependent*...
  - We were seeing exp(Mt), not the timeless object M
- Stimulus injection is artificial/ debatable

Zhiming's leap:

Estimating M^{-1} w/ *long, steady* CRM runs

Linear Response Functions of a Cumulus Ensemble to Temperature and Moisture Perturbations and Implications for the Dynamics of Convectively Coupled Waves

ZHIMING KUANG

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(Manuscript received 16 July 2009, in final form 24 November 2009)

ABSTRACT

An approach is presented for the construction of linear response functions of a cumulus ensemble to large-scale temperature and moisture perturbations using a cloud system–resolving model (CSRM). A set of time-invariant, horizontally homogeneous, anomalous temperature and moisture tendencies is added, one at a time, to the forcing of the CSRM. By recording the departure of the equilibrium domain-averaged temperature and moisture profiles from those of a control experiment and through a matrix inversion, a sufficiently complete and accurate set of linear response functions is constructed for use as a parameterization of the cumulus ensemble around the reference mean state represented by the control experiment.

Zhiming's inverse casting of problem

read off time-mean, domain mean sounding anoms.

$$\begin{bmatrix} T'(p) \\ q'(p) \end{bmatrix} = \left[\mathbf{M}^{-1}\right] \begin{bmatrix} \dot{T}_c'(p) \\ \dot{q}_c'(p) \end{bmatrix}$$

Bumps on forcing profile provoke bumps on CRM's time-mean convective heating/ drying profiles

- ^^ With these, build M^{-1} column by column
- Invert to get M! (computer knows how)
- *Test: reconstruct transient stimulus problem*

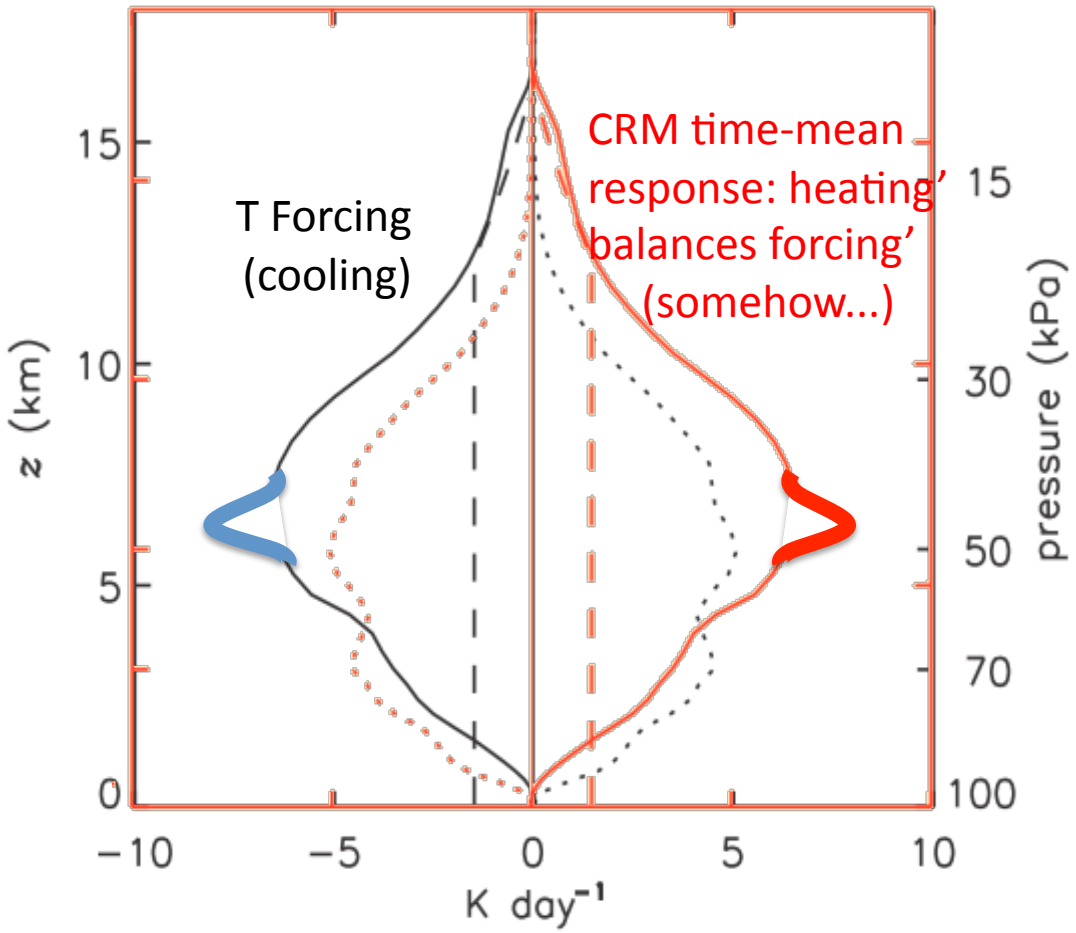
Outline

- Linearity (!) of deep convection (anomalies)
- The system in matrix form
  - build it in a spatial basis: CRM & inversion (ZK)

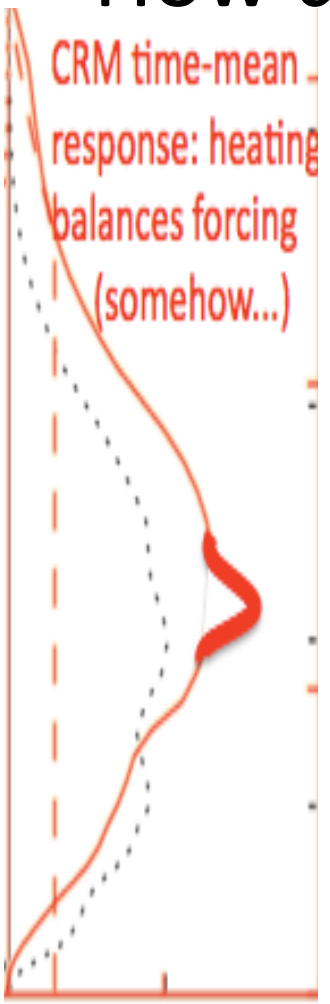
  - math checks, eigenvector/eigenvalue basis, etc.

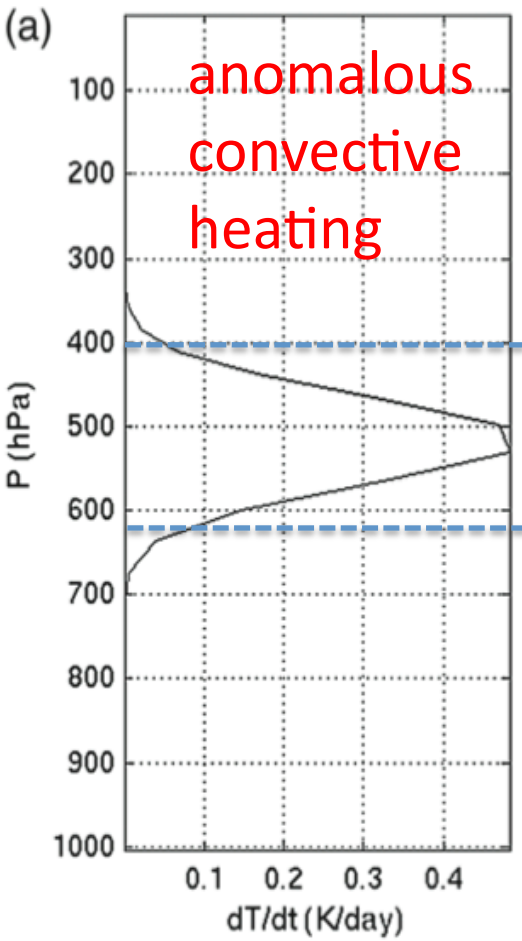
- Can we estimate M from obs?
  - (and model-M from GCM output in similar ways)?

Zhiming put a bump (perturbation) on the forcing profile, and studied the *time-mean response* of the CRM

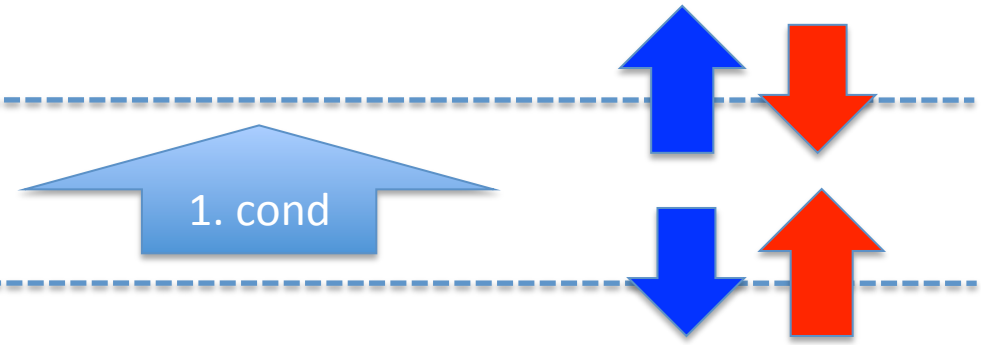


How does convection make a heating bump?





$$= \left[L/C_p (c - e) \right]_+ + \left[-\partial/\partial p (\omega' T') \right]$$



1. Extra Condensation in anomalous cloudy upward mass flux

2. EDDY: Updrafts/ downdrafts are extra warm/cool, *relative to environment*, in *quadrature* with heating bump

Now, how does convection know to do this?

How does convection 'know'? Env. tells it!

presumably by shaping buoyancy profiles $(T_{vp} - T_{ve})$

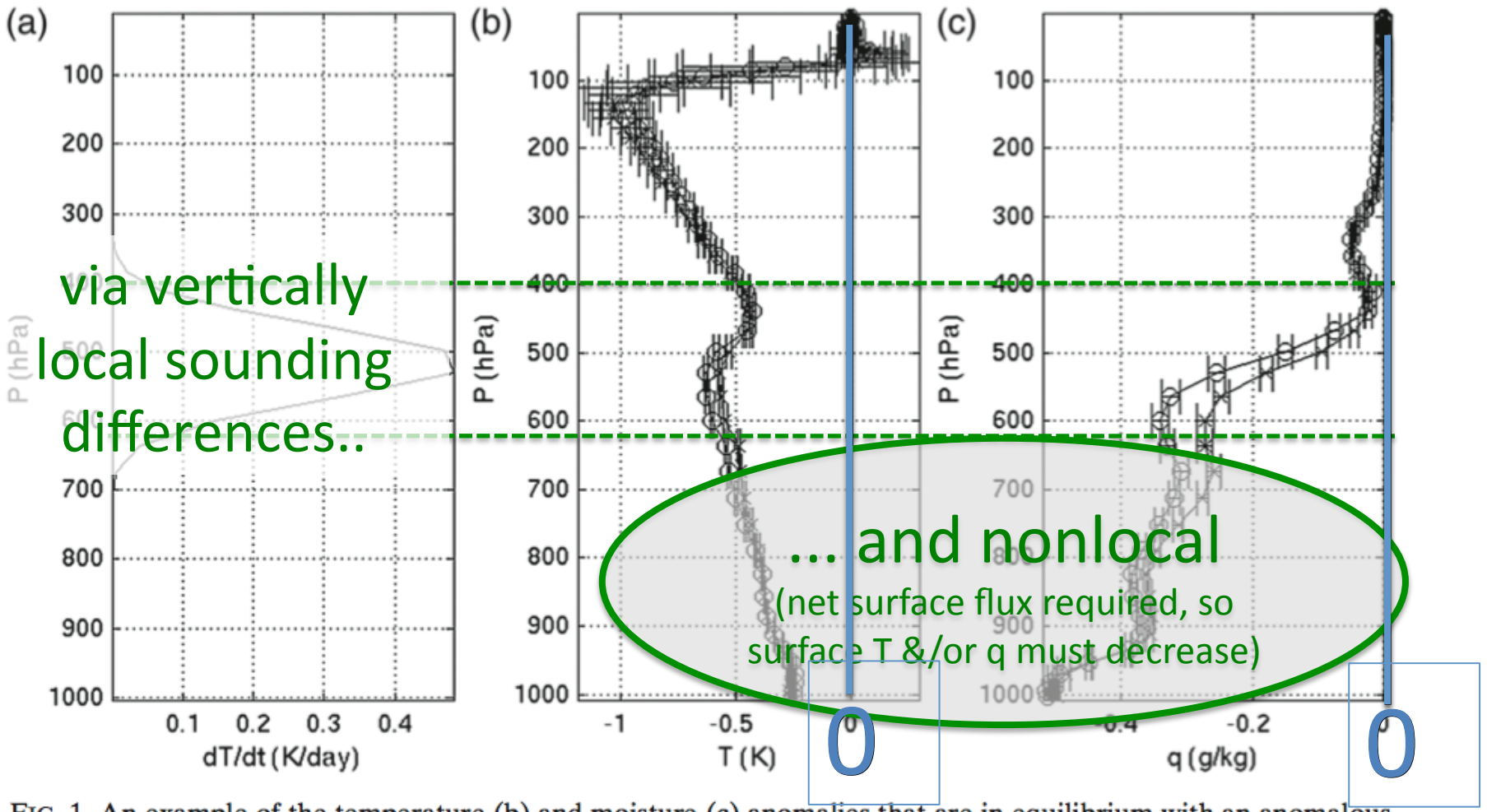
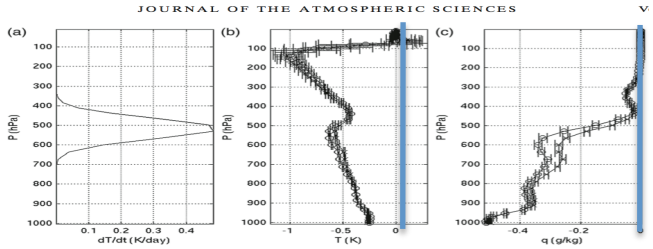


FIG. 1. An example of the temperature (b) and moisture (c) anomalies that are in equilibrium with an anomalous convective heating profile shown in (a) and zero convective moistening tendencies everywhere.

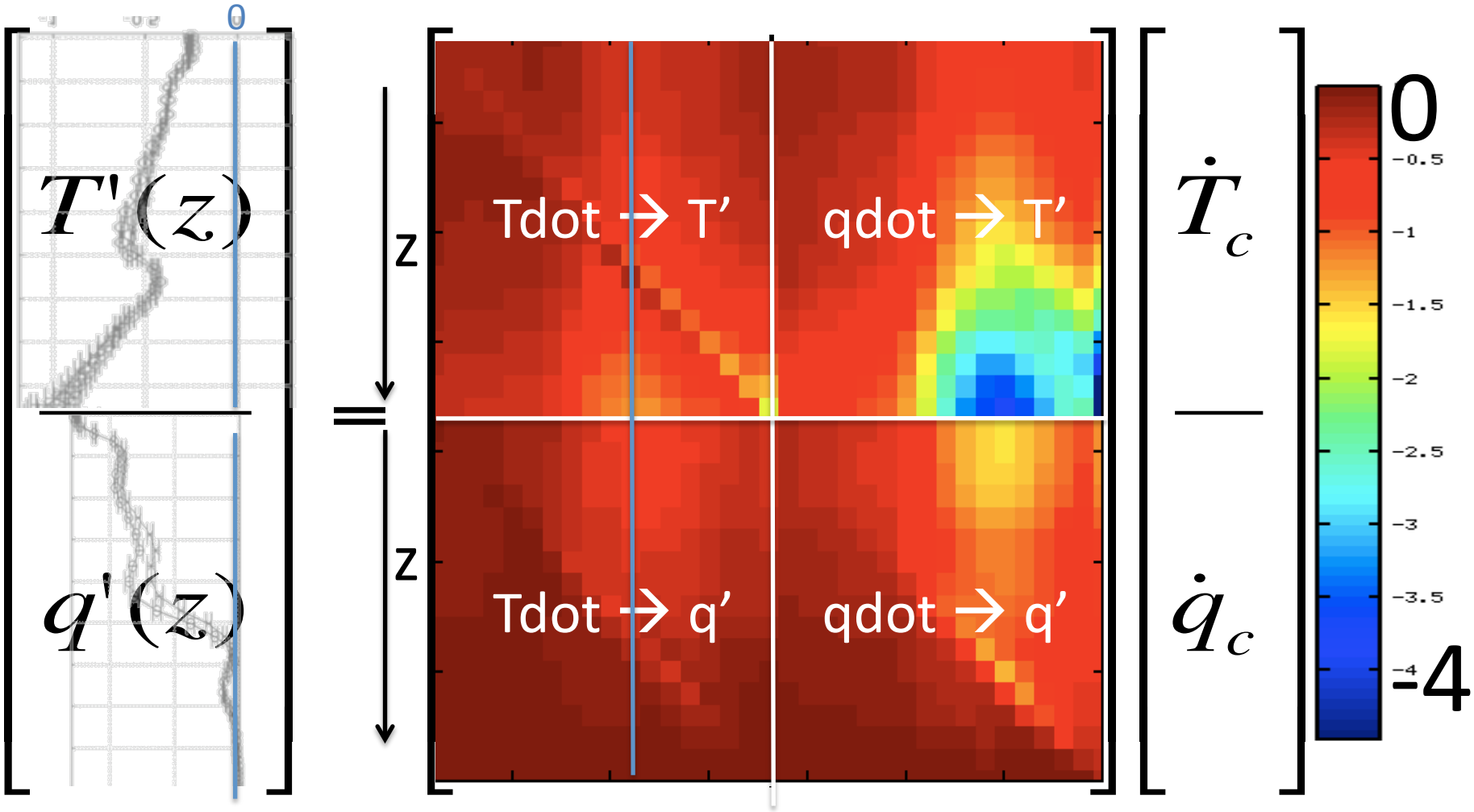
Kuang 2010 JAS



←It's a column of M^{-1}

$$\begin{bmatrix} T'(p) \\ \hline q'(p) \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ \hline 0 \\ 0 \\ 0 \end{bmatrix}$$

Image of M^{-1}

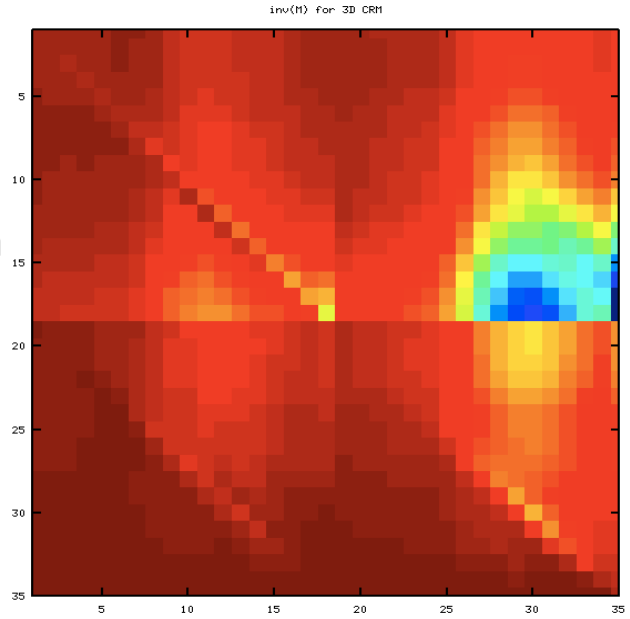


Units: K and g/kg for T and q, K/d and g/kg/d for heating and moistening rates

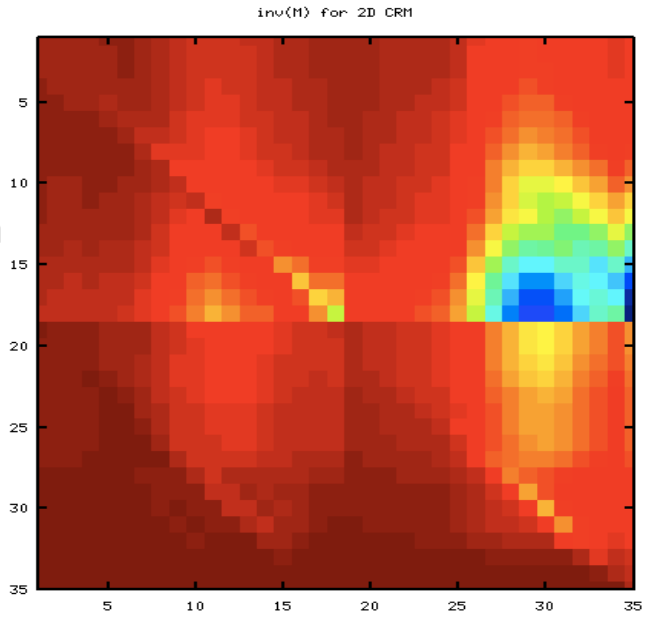
Unpublished matrices in model stretched z coordinate, courtesy of Zhiming Kuang

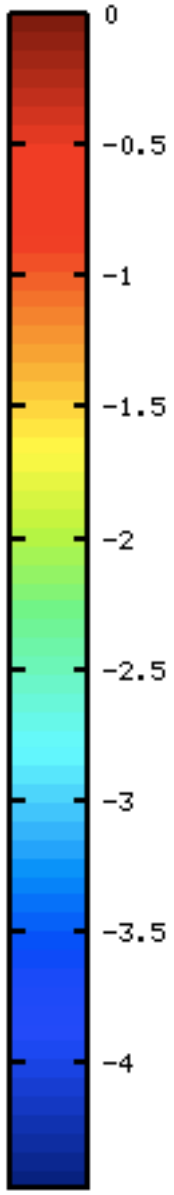
M^{-1} and M

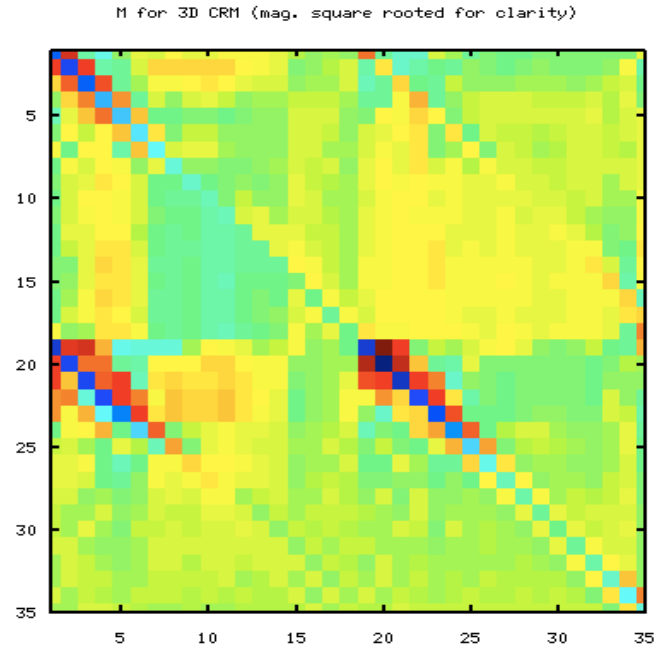
from 3D CRM

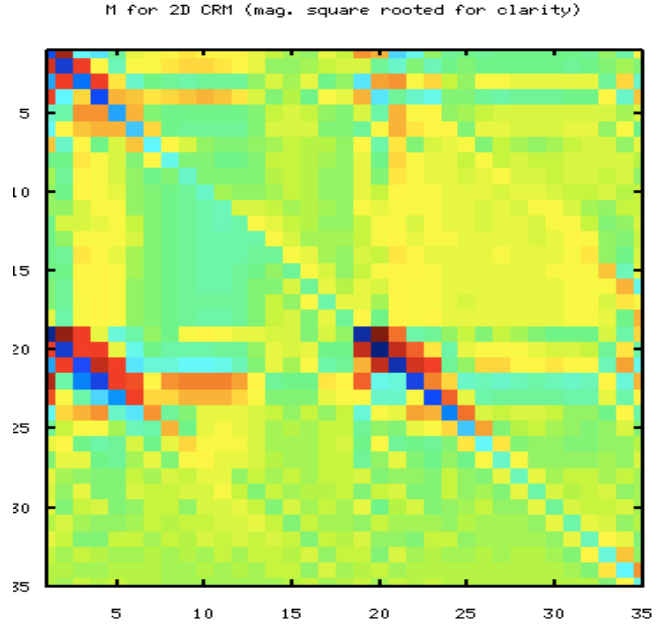


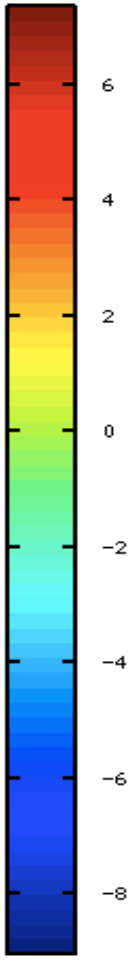
from 2D CRM







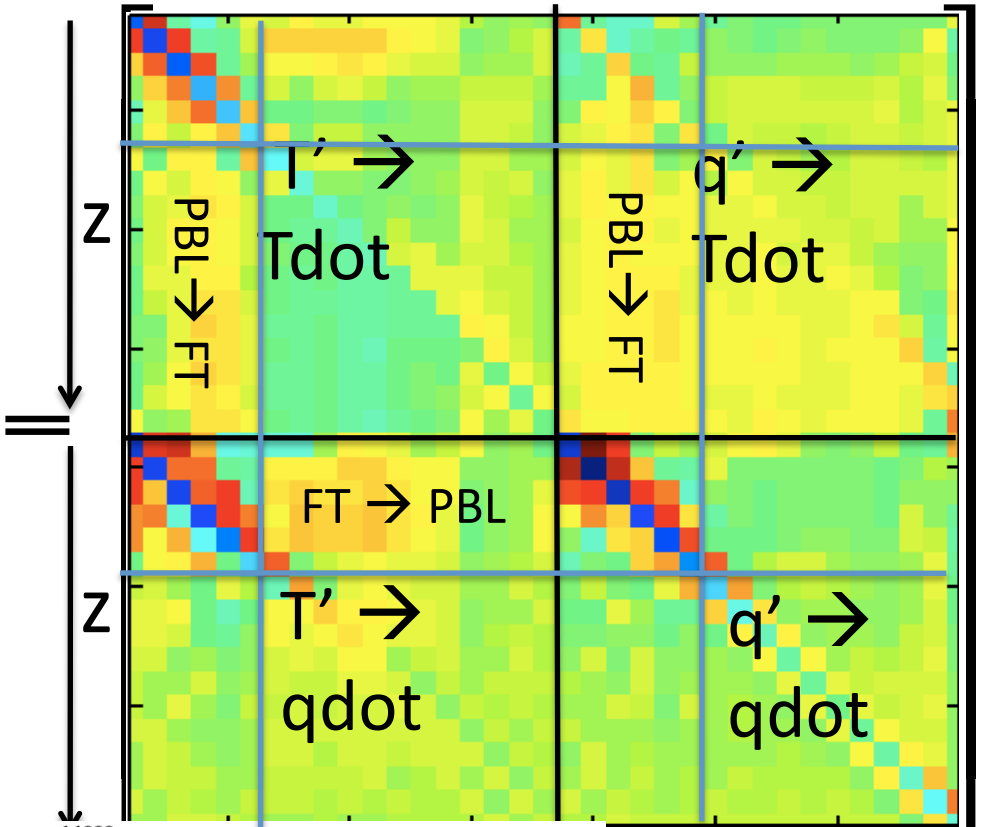


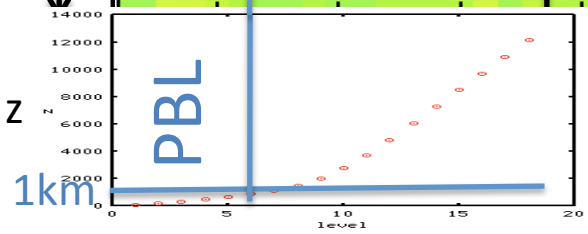


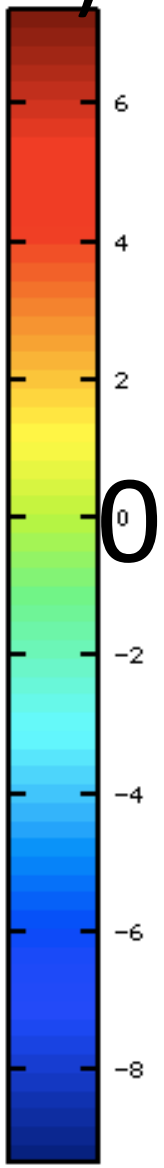
(^^^ these numbers are the sign coded *square root* of $|M_{ij}|$ for clarity of small off-diag elements)

Image of M (stretched z coords, 0-12km)

$$\begin{bmatrix} \dot{T}_c(z) \\ \hline \dot{q}_c(z) \end{bmatrix} = \begin{bmatrix} T \\ \hline q \end{bmatrix}$$







square root color scale

Image of M (stretched z coords, 0-12km)

Units: K and g/kg for T and q, K/d and g/kg/d for Tdot and qdot

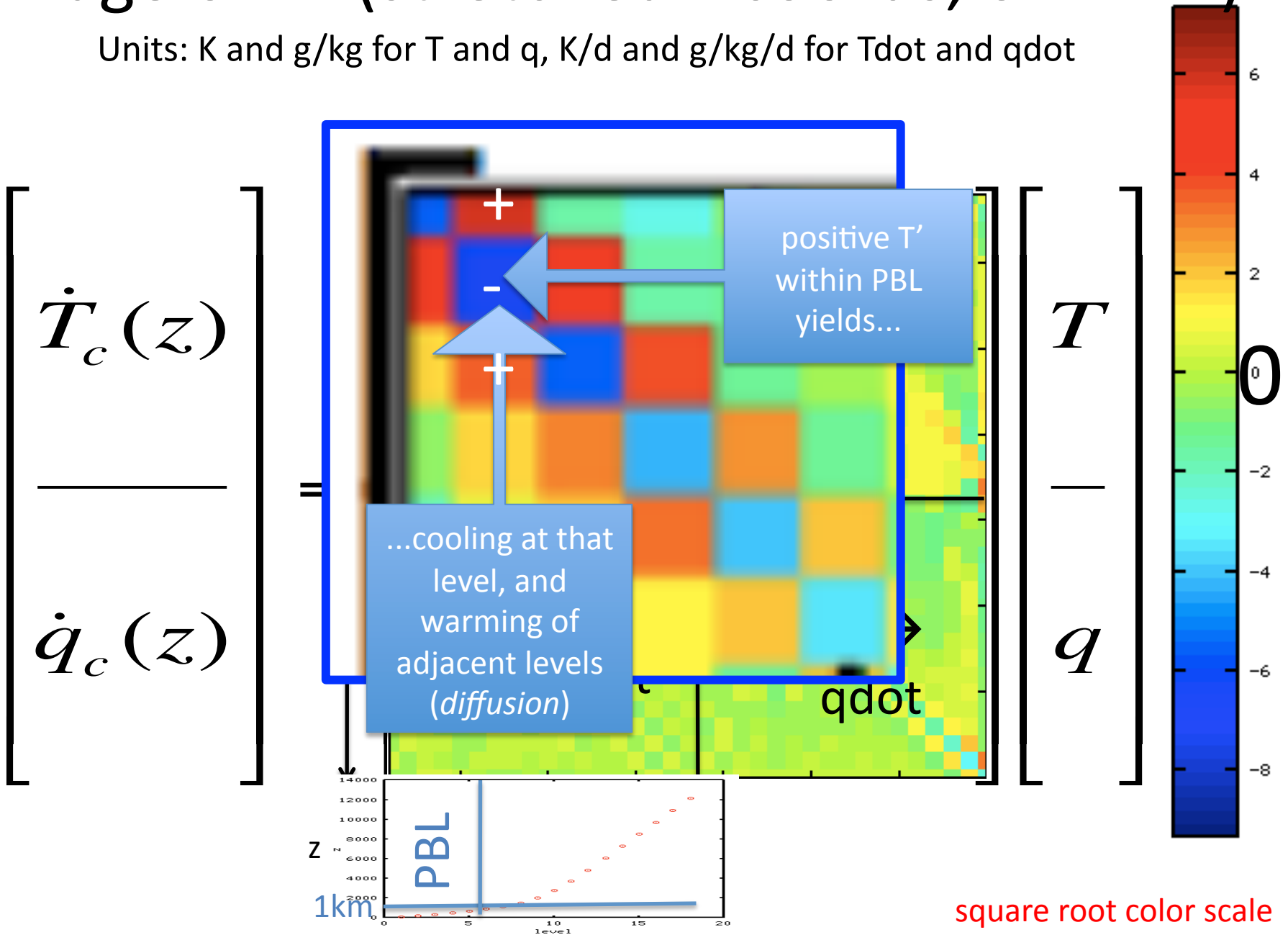
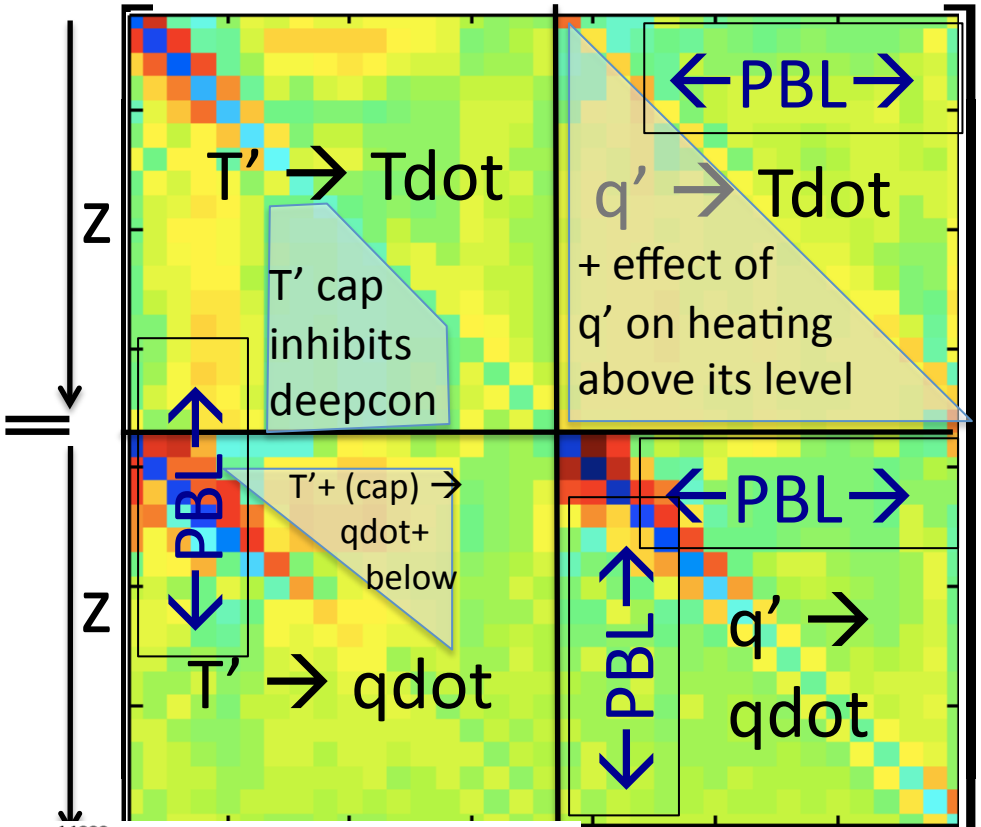
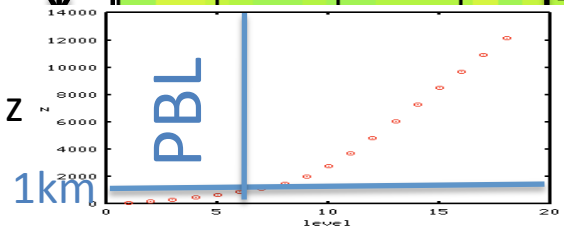


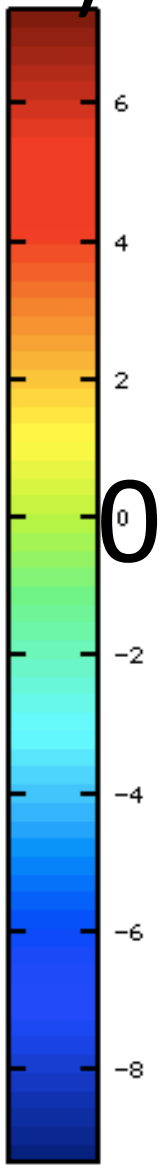
Image of M (stretched z coords, 0-12km)

Units: K and g/kg for T and q,
K/d and g/kg/d for Tdot and qdot

$$\begin{bmatrix} \dot{T}_c(z) \\ \hline \dot{q}_c(z) \end{bmatrix} = \begin{bmatrix} T \\ \hline q \end{bmatrix}$$







square root color scale

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- Linearity (!) of deep convection (anomalies)
- The system in matrix form
  - mapped in a spatial basis, using CRM & inversion (ZK)
  - eigenvector/eigenvalue basis

- Can we estimate M from obs?
  - (and model-M from GCM output in similar ways)?

Eigenvectors: a new basis

$$[\mathbf{M}]\mathbf{v}_j = \lambda_j \mathbf{v}_j$$

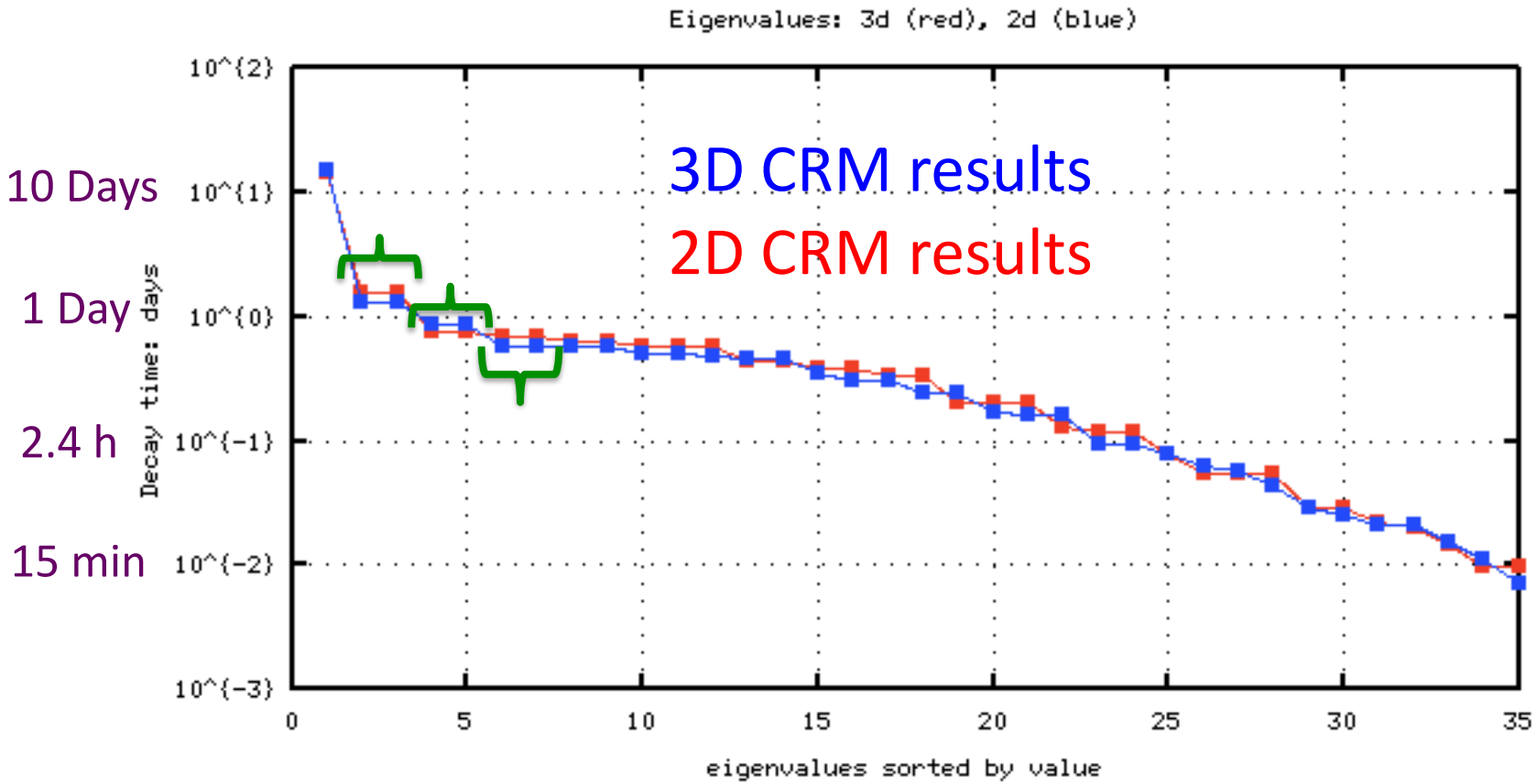
There are j → 35 independent (but not orthogonal) eigenvectors (each with an eigenvalue) for this well-behaved 35x35 matrix M (18 T levels, 17 q levels).

- *The real parts of all 35 eigenvalues are negative*: because there are *no unstable, growing T' or q' profile structures* in a steadily forced convecting CRM.

- *The imaginary parts can be anything*: both positive and negative λ_{im} represent the same oscillation in real profiles T(z), q(z), so both appear (in c.c. pairs of λs)

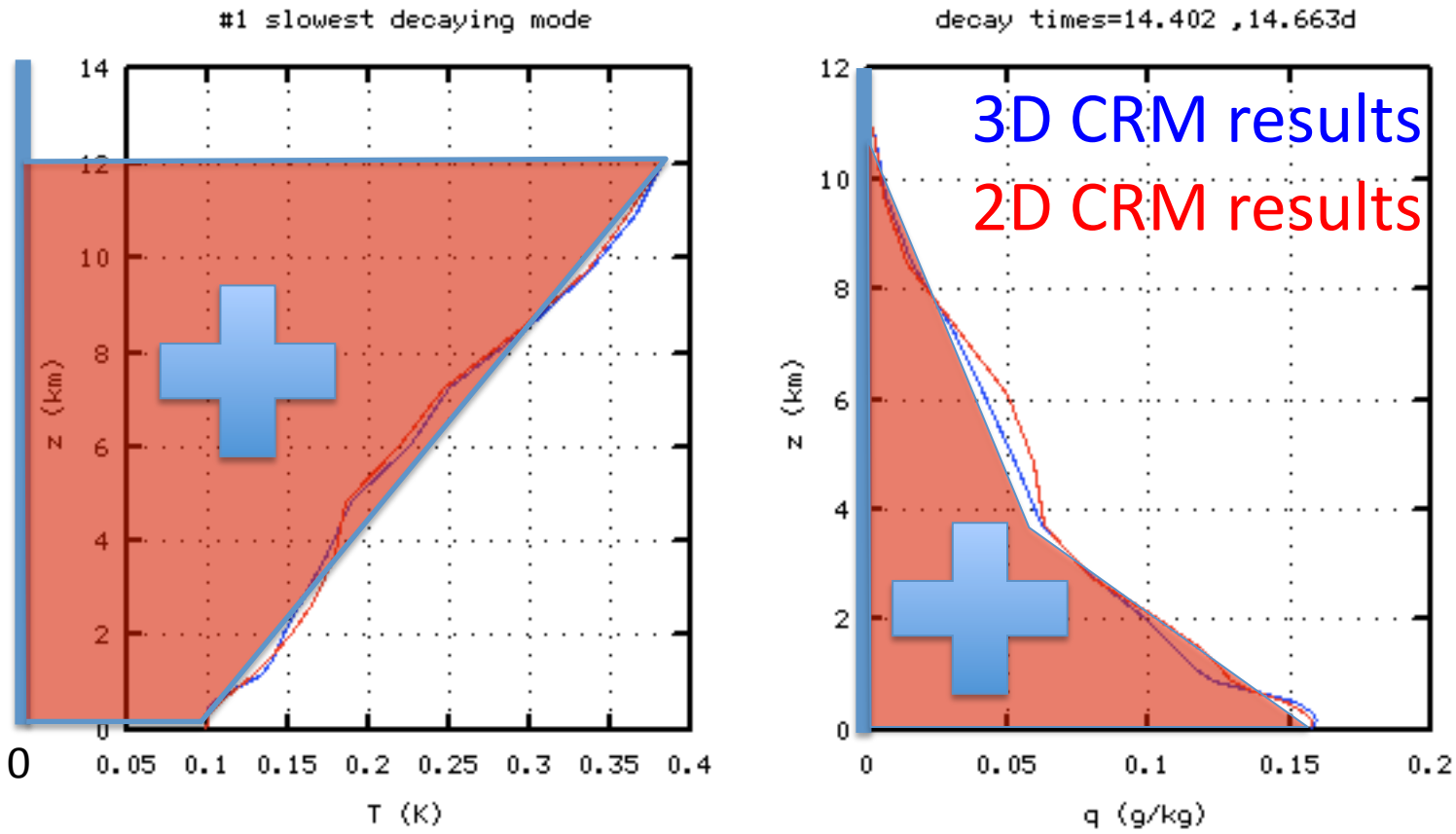
The 35 Eigenvalues

Sorted according to inverse of real part (**decay timescale**)



There is one mode with slow (2 weeks!) decay,
a few complex conj. pairs for o(1d) decaying oscillations,
"the rest" (e.g. diffusion damping of PBL T', q' wiggles)

Gravest eigenvector: 14d decay time

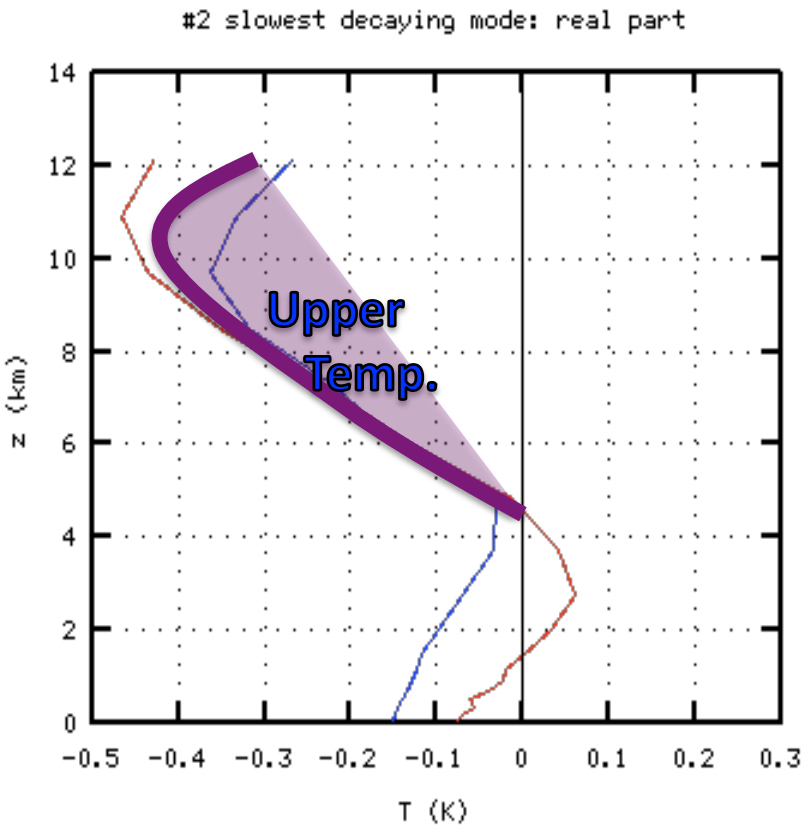


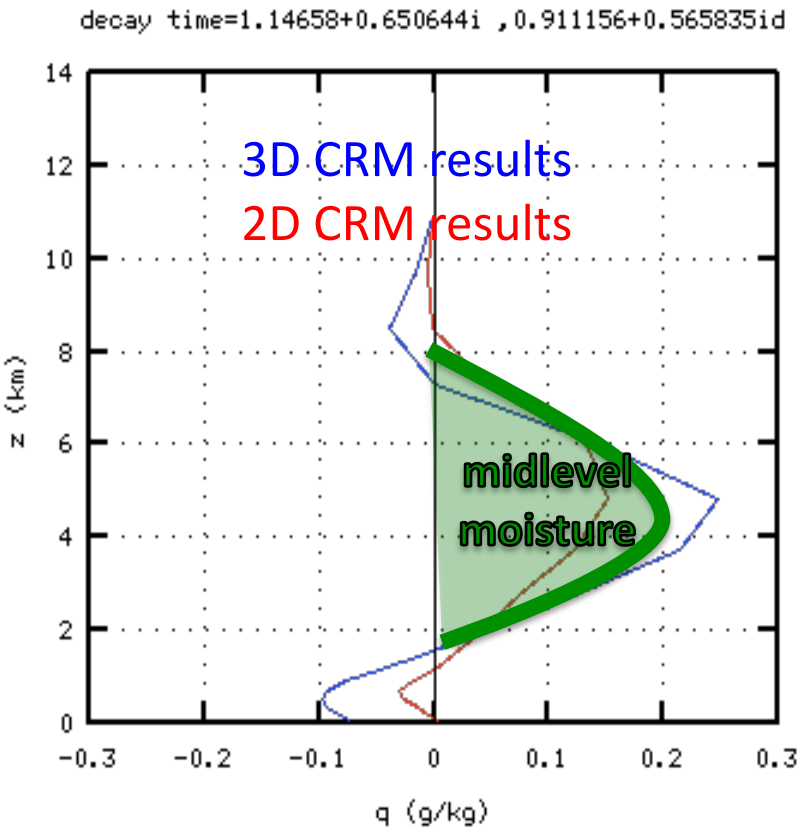
Column MSE anomalies damped *only* by surface flux anomalies, *with fixed wind speed in flux formula*.

A CRM setup artifact.*

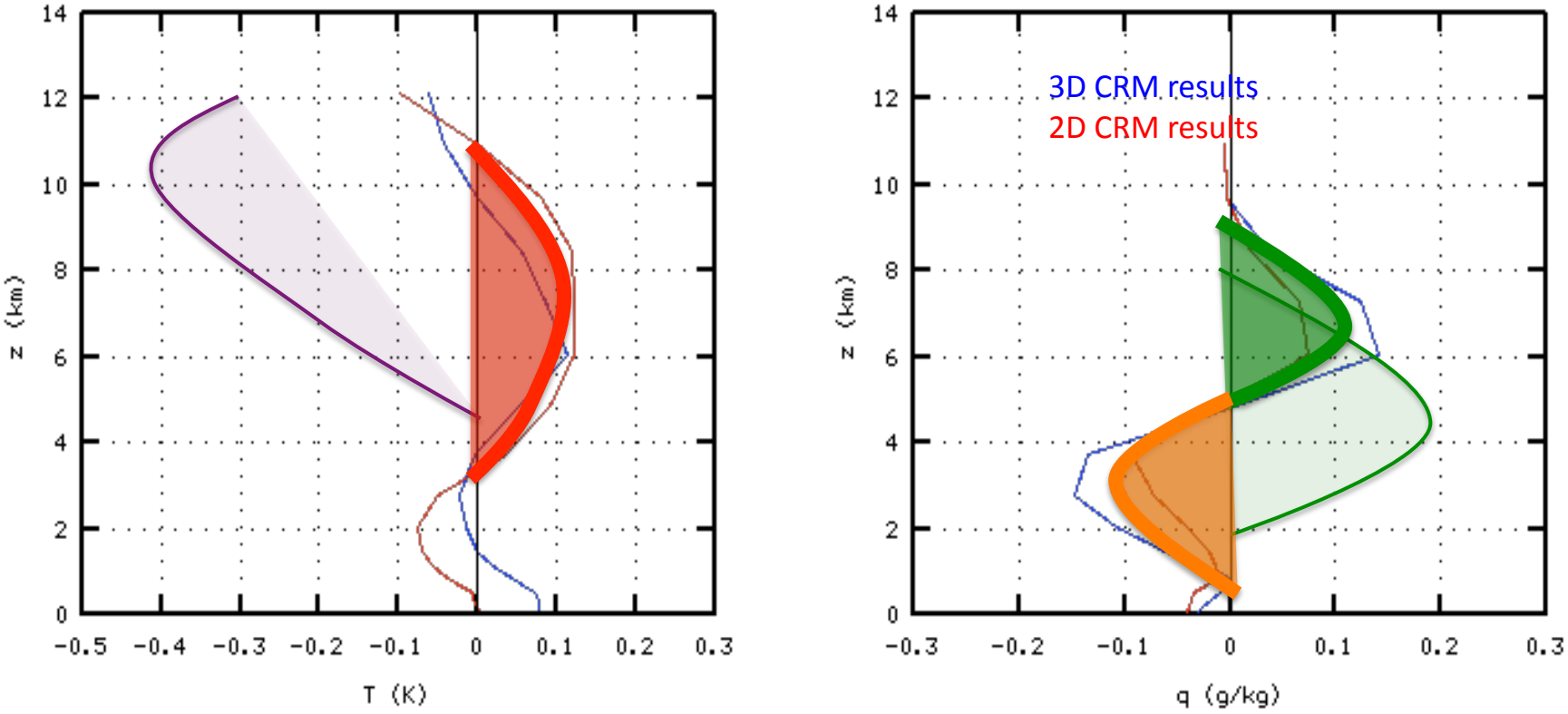
*(But T/q relative values & shapes have info about moist convection?)

eigenvector pair #2 & #3. Phase: 0^{o}
~1d decay time, ~2d osc. period





90^{o} phase: moist layer has ascended, net heating (aloft) & drying (at 2-4km) ~1d decay time, ~2d osc. period



Interp: **congestus-deep convection oscillations**

this phase (moist & unstable midlevels) causes tendencies toward its opposite (upper warm cap over dried midlevels...which then moisten... etc.)

(Zhiming verified oscillations by transplanting this structure into CRM)

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- Linearity (!) of deep convection (anomalies)
- The system in matrix form
  - mapped in a spatial basis, using CRM & inversion

    anything a CRM can do, an SCM can do
  - math checks, eigenvector/eigenvalue basis, etc.

- Can we estimate M from obs?
  - (and model-M from GCM output, in similar way)?
    - an important role for GCMs

Exploiting the Information Content of obs (or GCM simulation output)

- Link T/q to conv. heating/drying via composites
  - they tend to show a statistically steady state
    - times of increase and times of decrease are averaged
  - *Might the time-sequence-blurring properties of averaged composites be a strength ??*
    - like in Zhiming's steady CRM approach?

Exploiting the Information Content of obs (or GCM simulation output)

- A few challenges
  - Variables: we don't measure heating and drying
    - main observable w/ info content is *horiz. wind divergence*
  - Coordinates: can't control level by level (z or p)
    - weakly-damped modes a more natural basis
      - observed at significant amplitude, hence with less error

Proposal: a data-attuned activity to characterize convection via **M**

- Build matrix in obs-friendly variables
  - div(p)
    - *observable*; w(p) is linearly related but spreads errors
  - Lapse rate profile Γ (p), plus T_{surf} for completeness
    - observable; more physically related to convection than local T(p)
  - q(p) -- since moisture is locally linked to convection
- Build **M** in the most obs-error-robust basis
  - least damped modes (CRM eigenvecs as first guess)
    - rather than level by level

Proposal: a data-attuned activity to characterize convection via **M**

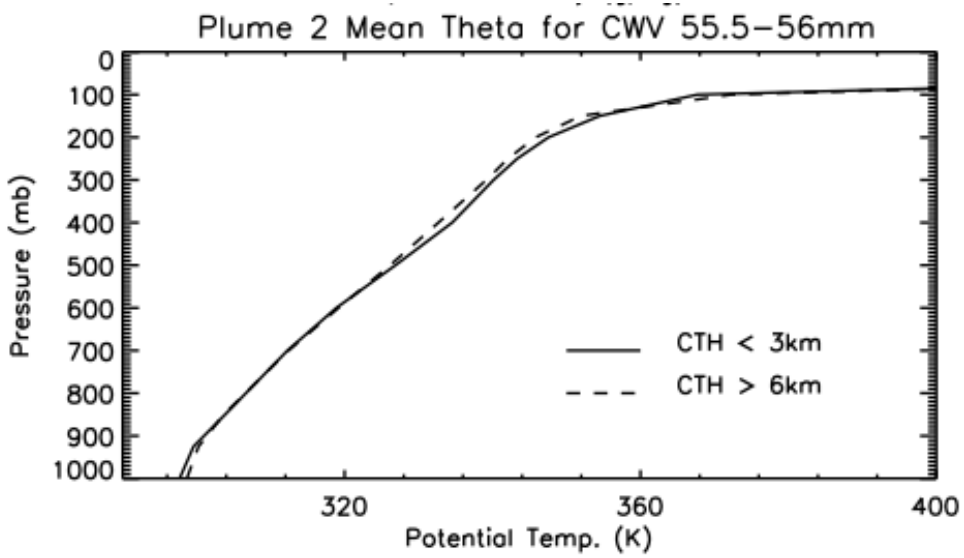
- Once we have a robust matrix,
  - transforming variables is straightforward
    - e.g. div' → w'
    - or transform Γ back to T if desired
  - transforming coordinates is linear
    - find eigenvectors; iterate if different from CRM
- Application can be in the most powerful, incisive, useful basis of variables and coordinates

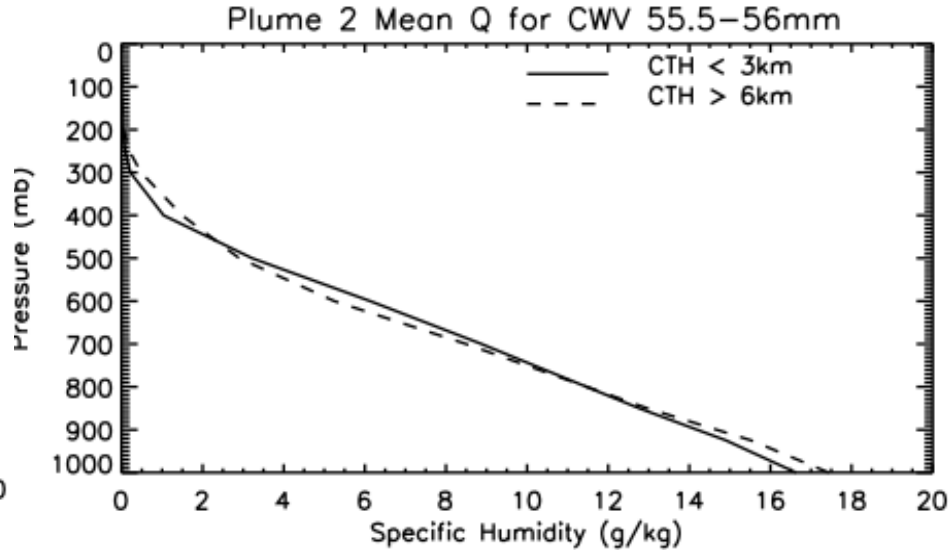
Example 1: use of M to evaluate GCM

- **THE MJO TRANSITION FROM SHALLOW TO DEEP CONVECTION IN CLOUDSAT/CALIPSO DATA AND GISS GCM SIMULATIONS**
  - **Anthony D. Del Genio, Yonghua Chen, Daehyun Kim, Mao-Sung Yao**
    - **Journal of Climate Submitted July 1, 2011**

Del Genio et. al. 2011 GISS GCM result

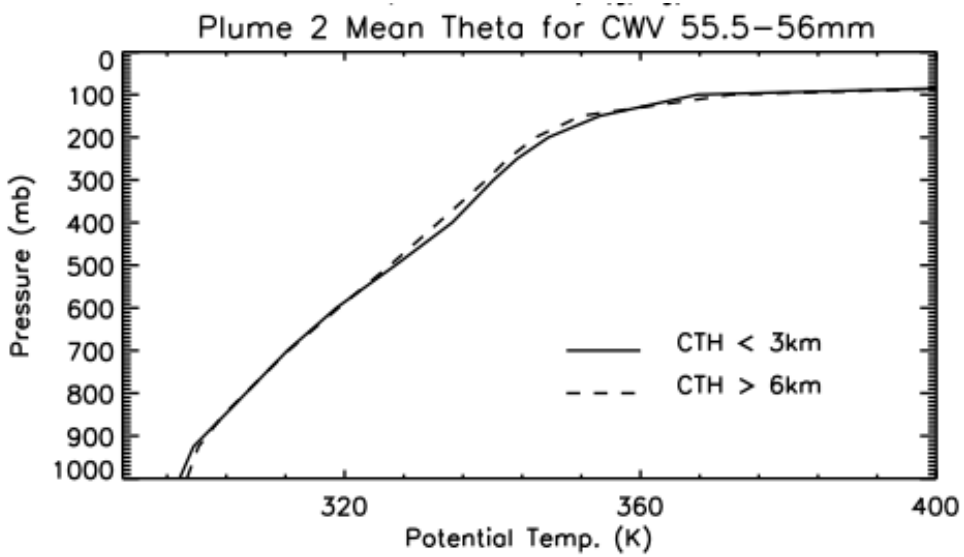
- GISS GCM columns with same WVP (PW)
  - » but different plume tops (<3km, >6km)

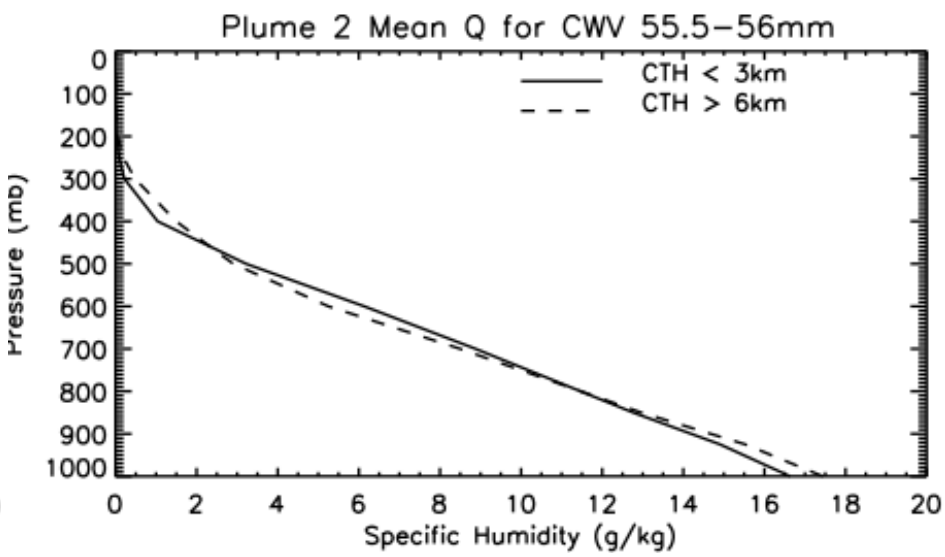


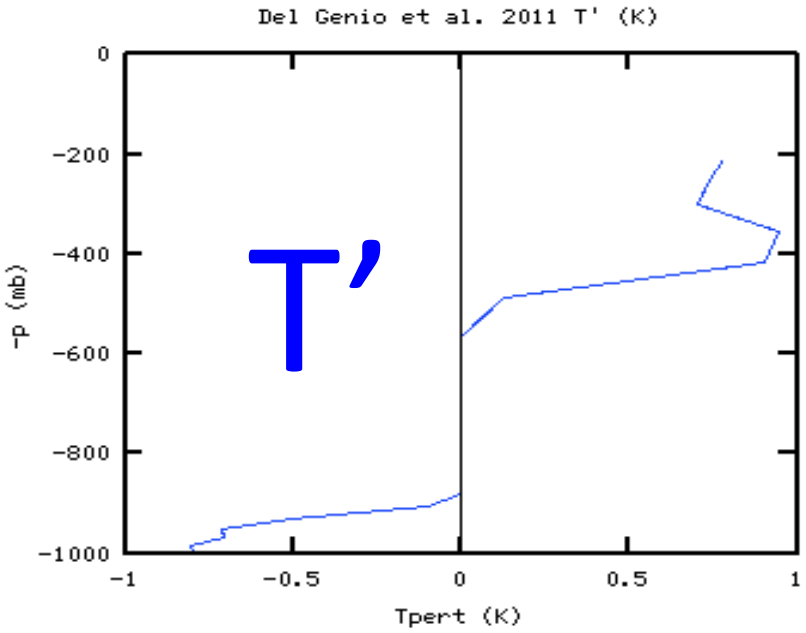


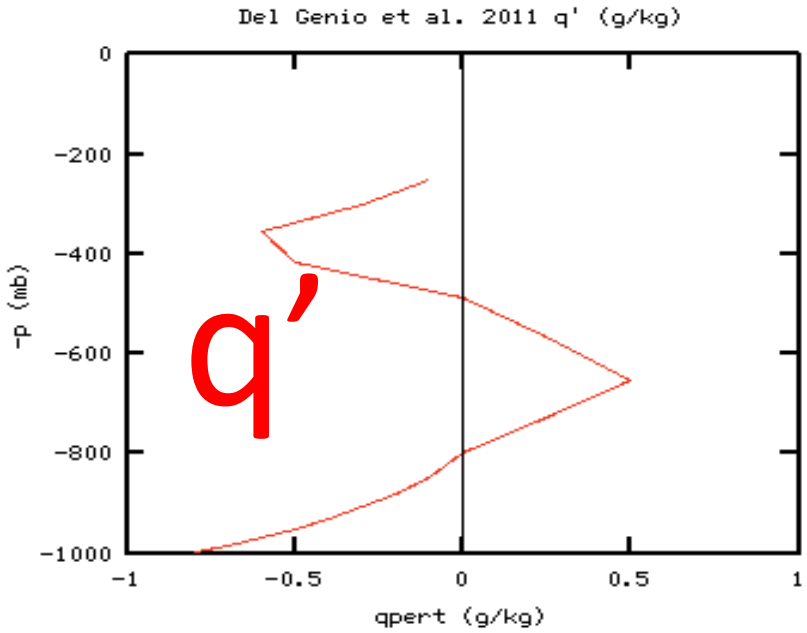
- Is the GCM's response to this T', q' appropriate?
- Which should be more important: T' or q'?

Del Genio et. al. 2011 GISS GCM result

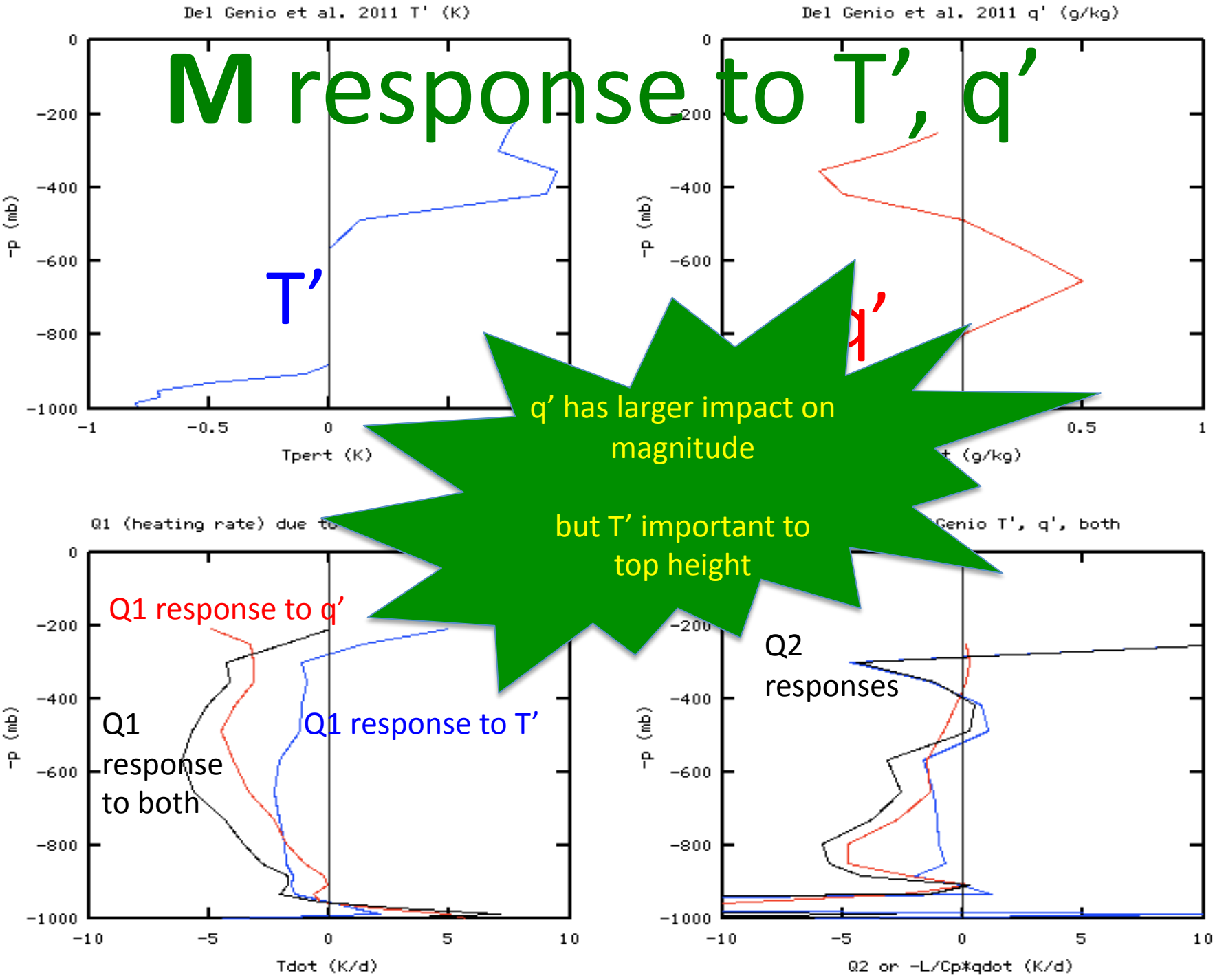








M response to T’, q’

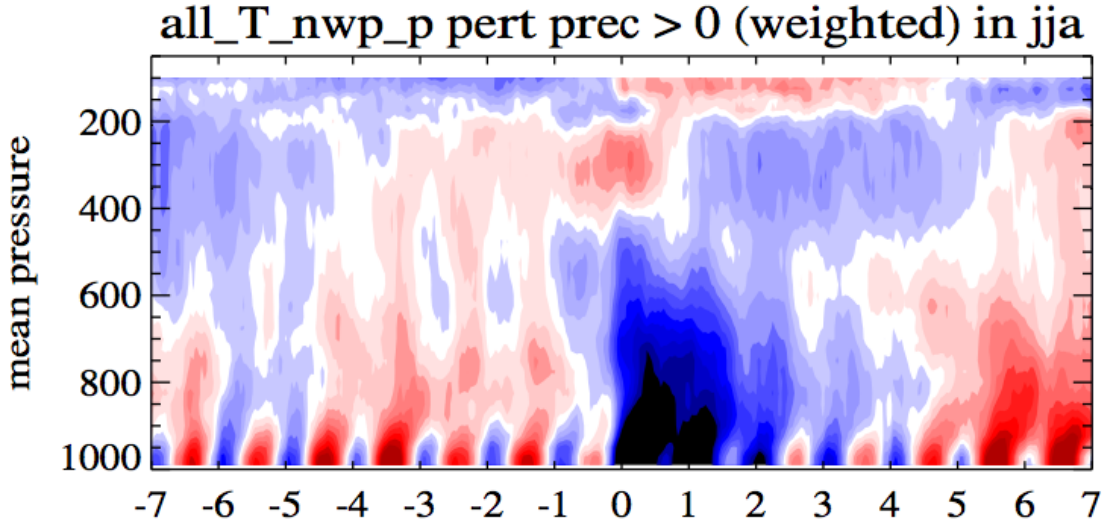


Example 2: ARM data to build $\mathbf{M}^{-1}$?

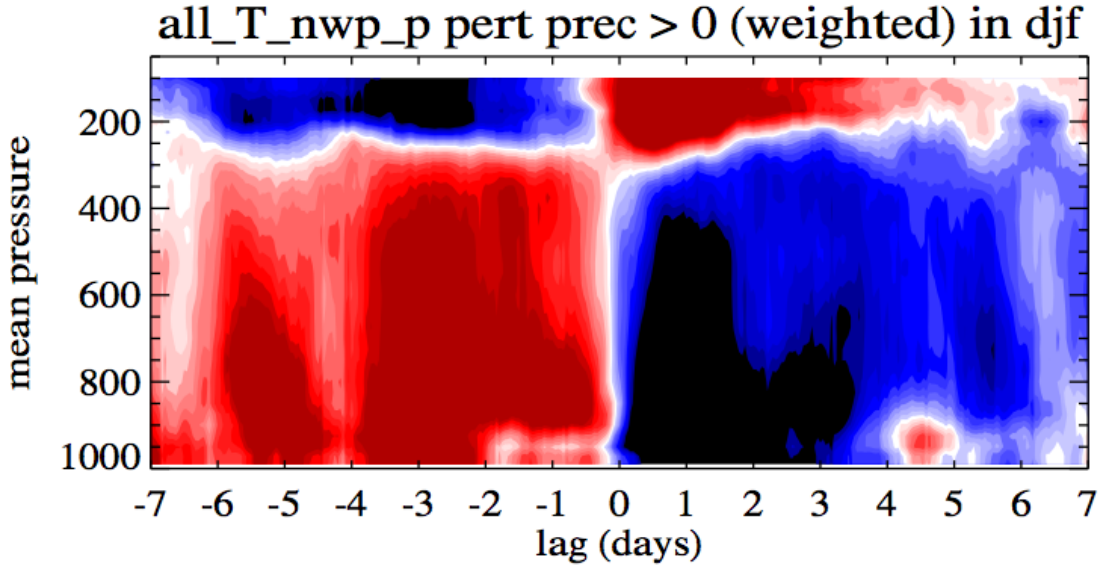
- **Composites of CMBE T' and q' around rain events in summer vs. winter at SGP**
  - Emily Riley, Siwon Song, Brian Mapes
    - » In preparation
- Can we stack these into column vectors meaningfully?
- Can we use the lag information sensibly?

Seasonal:

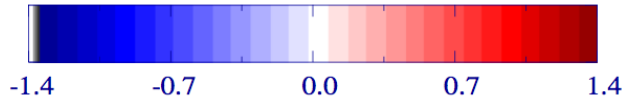
Temperature perturbation [K]



JJA: Summer

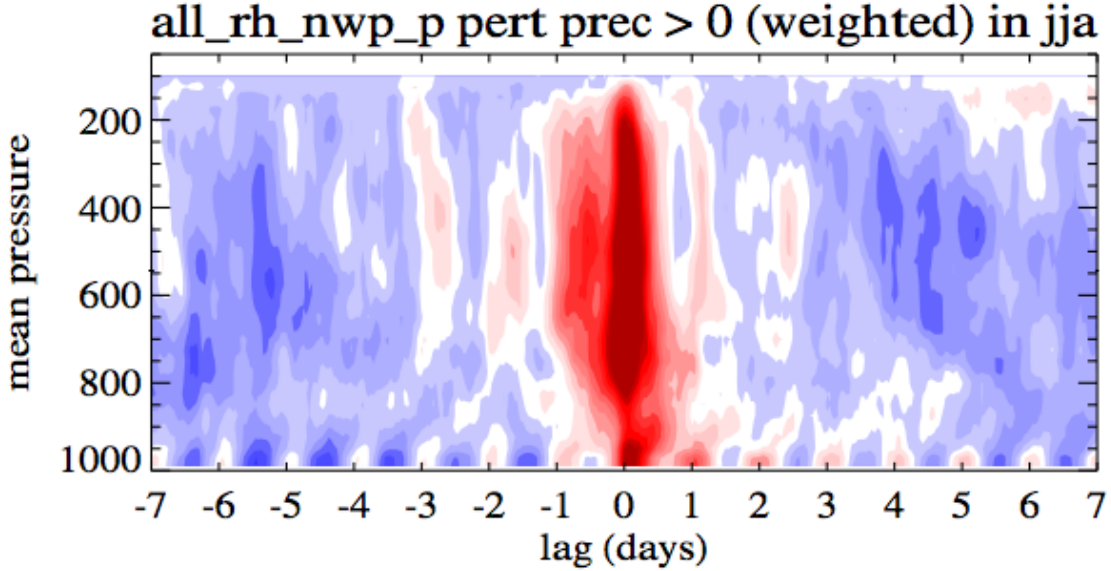


DJF: Winter

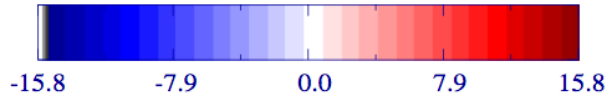


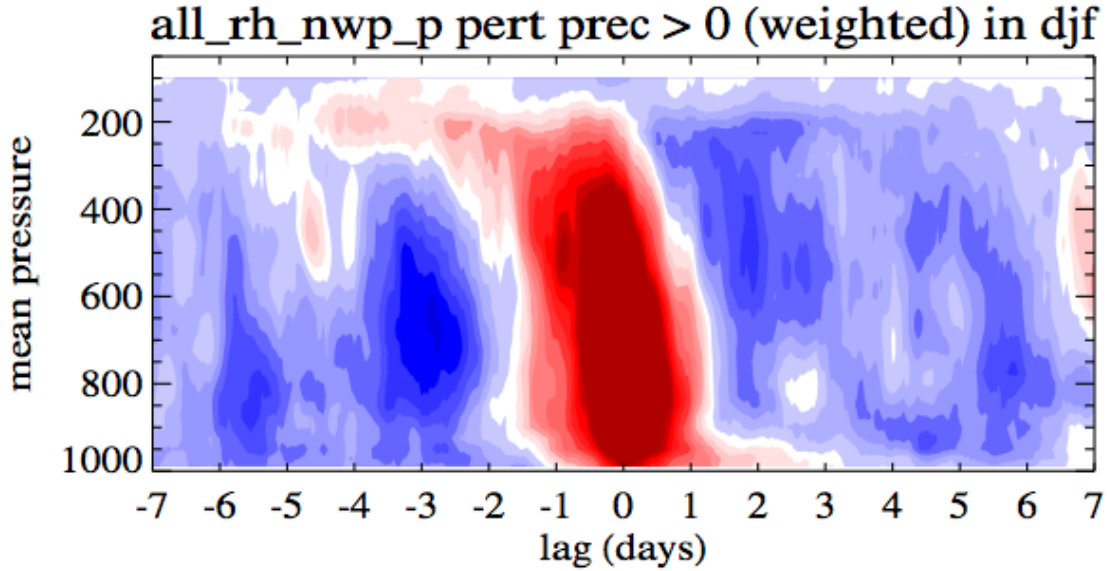
Seasonal:

Relative Humidity perturbation [%]

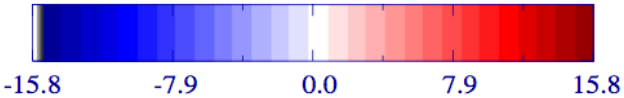


JJA: Summer





DJF: Winter



Wrapup

- Convection is a linearizable process
  - around convecting base states
    - How many? as many as give *dynamically distinct* M's.
- This unleashes a lot of math capabilities
  - e.g. basis/variable transformations are easy
    - for maximum convenience and then power
- ZK builds timeless **M** using *steady state* runs
  - can we do same w/obs. composites of T' and q' ?
    - around moist events (rain, heating, divergence...)
- Checks on the method are several
  - as are incisive obs/model comparisons
    - needs more work, more people on the same page!
    - Too big for one or two of us.
      - Lot of background needed though...