

# Relationships between Raindrop Size Distributions Parameters: Framework to Investigate Assumptions in Model Parameterizations

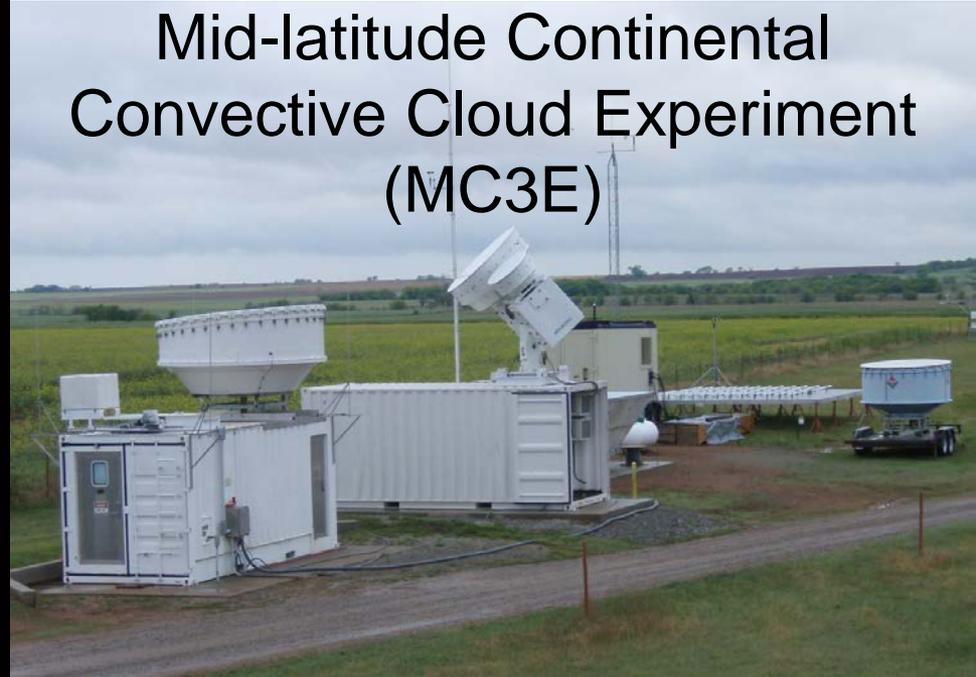
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Research in Environmental  
Sciences (CIRES)*



University of Colorado  
Boulder

## Mid-latitude Continental Convective Cloud Experiment (MC3E)



Support for this work:

DOE ASR Grant: DE-SC0007080

NASA PMM Grant: NNX13AF89G



# Bridging Observations & Algorithms

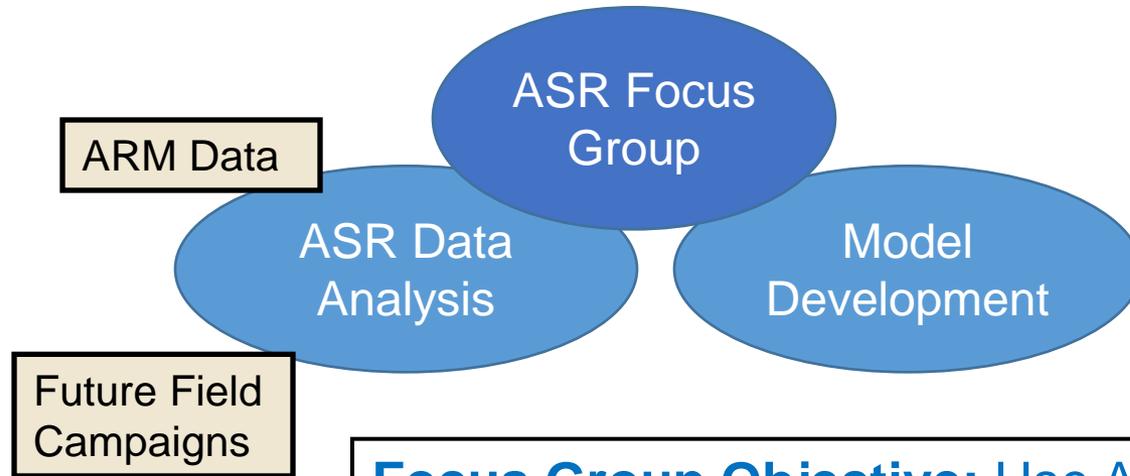
NASA Global Precipitation Mission (GPM) DSD Working Group:  
Bridging Ground Validation (GV) and Algorithms



**DSD WG General Objective:** Use Ground Validation (GV) data to support, or justify, **assumptions** used in satellite retrieval algorithms.

# Bridging Observations & Models

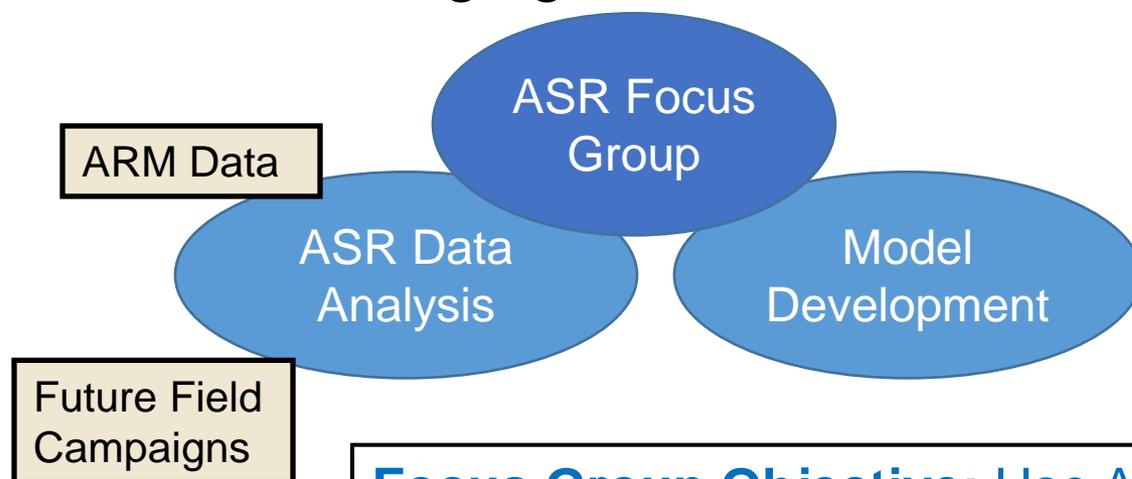
NASA Global Precipitation Mission (GPM) DSD Working Group:  
Bridging ARM Data and Models



**Focus Group Objective:** Use ARM Data to support, or justify, **assumptions (aka parameterizations)** used in models.

# Bridging Observations & Models

NASA Global Precipitation Mission (GPM) DSD Working Group:  
Bridging ARM Data and Models



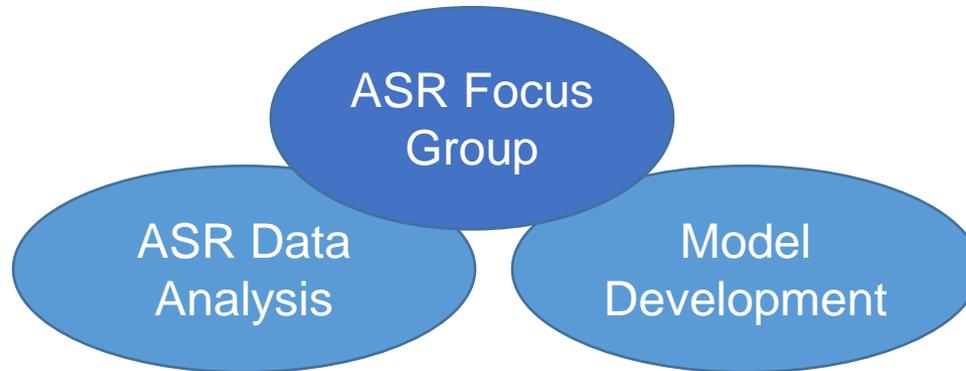
**Focus Group Objective:** Use ARM Data to support, or justify, **assumptions (aka parameterizations)** used in models.

## **Example of using Data to investigate assumptions:**

1. Monday (MC3E/Meso) - Relationships between DSD attributes.
2. Tuesday (ICEPROP) - Framework incorporating findings into Algorithms.

# Bridging Observations & Models

NASA Global Precipitation Mission (GPM) DSD Working Group:  
Bridging ARM Data and Models



***How are Raindrop Size Distributions (DSDs) described?:***

**Radar & Observations**

- 3 parameter gamma function  
( $N_w$ ,  $D_m$ ,  $\mu$ )

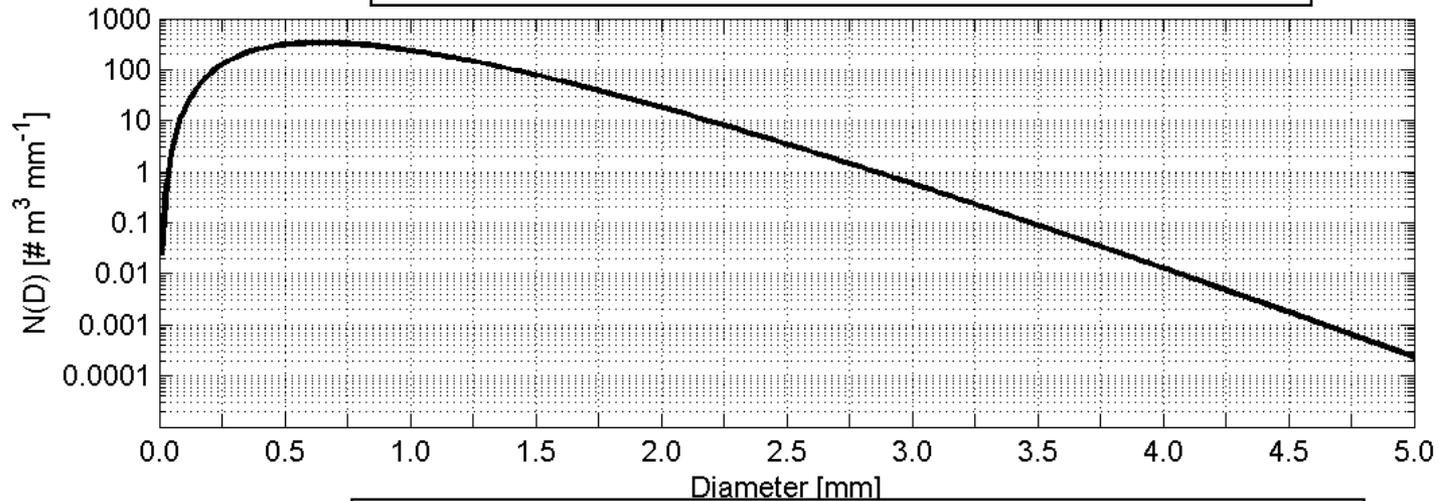
**Cloud Models**

- single moment
- double moment
- bin microphysics

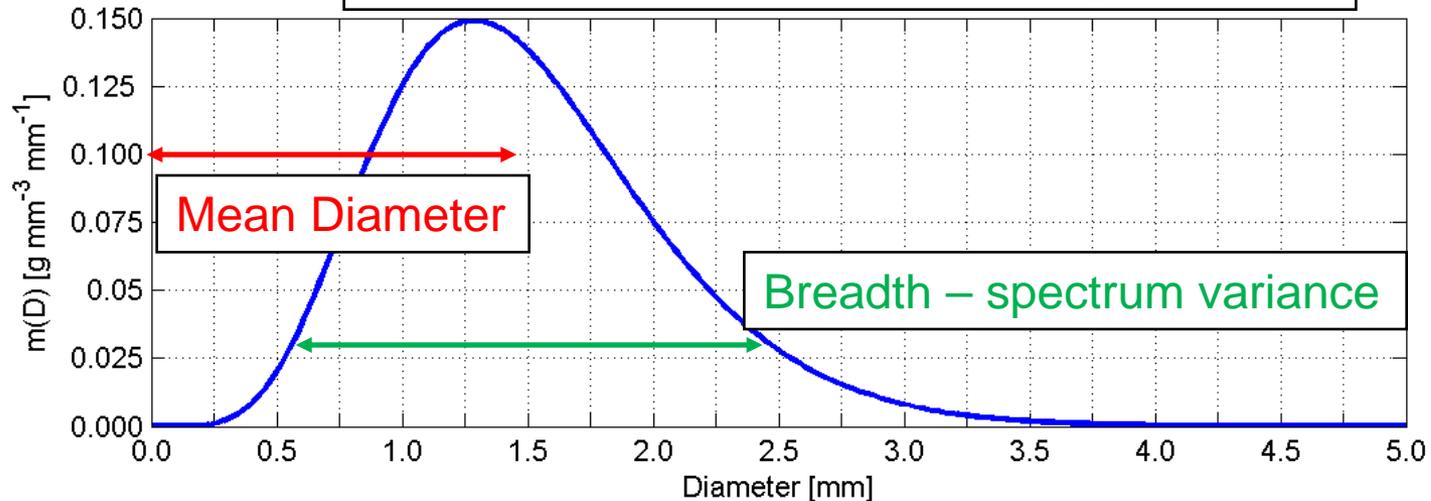
*Difficult to compare results because of different descriptions.*

# DSD Mass Spectrum

Number Concentration,  $N(D)$  [ $\# \text{ m}^{-3} / \text{mm}$ ]



Mass Spectrum,  $m(D) \sim N(D) D^3$  [ $\text{g m}^{-3} / \text{mm}$ ]



# DSD Mass Spectrum

## Mass Spectrum

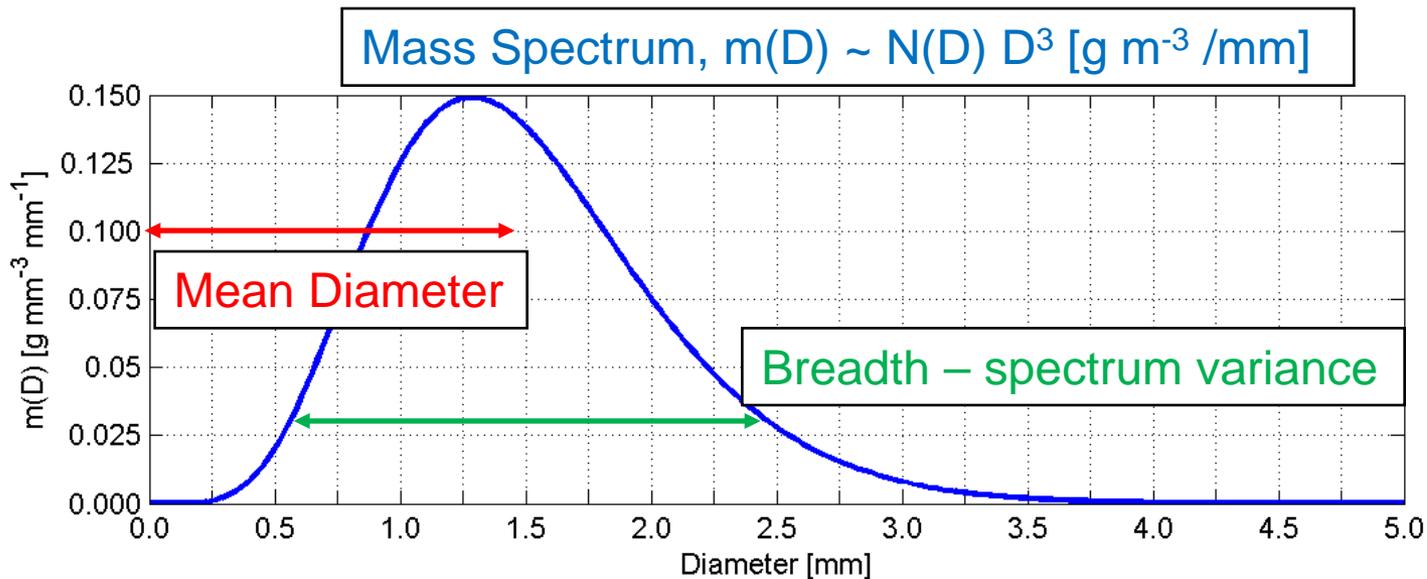
$$w(D) = \frac{\pi}{6 \cdot 10^3} \rho_w N(D) D^3$$

## Mean Diameter

$$D_m = \frac{\sum_{D_{\min}}^{D_{\max}} w(D) D dD}{\sum_{D_{\min}}^{D_{\max}} w(D) dD}$$

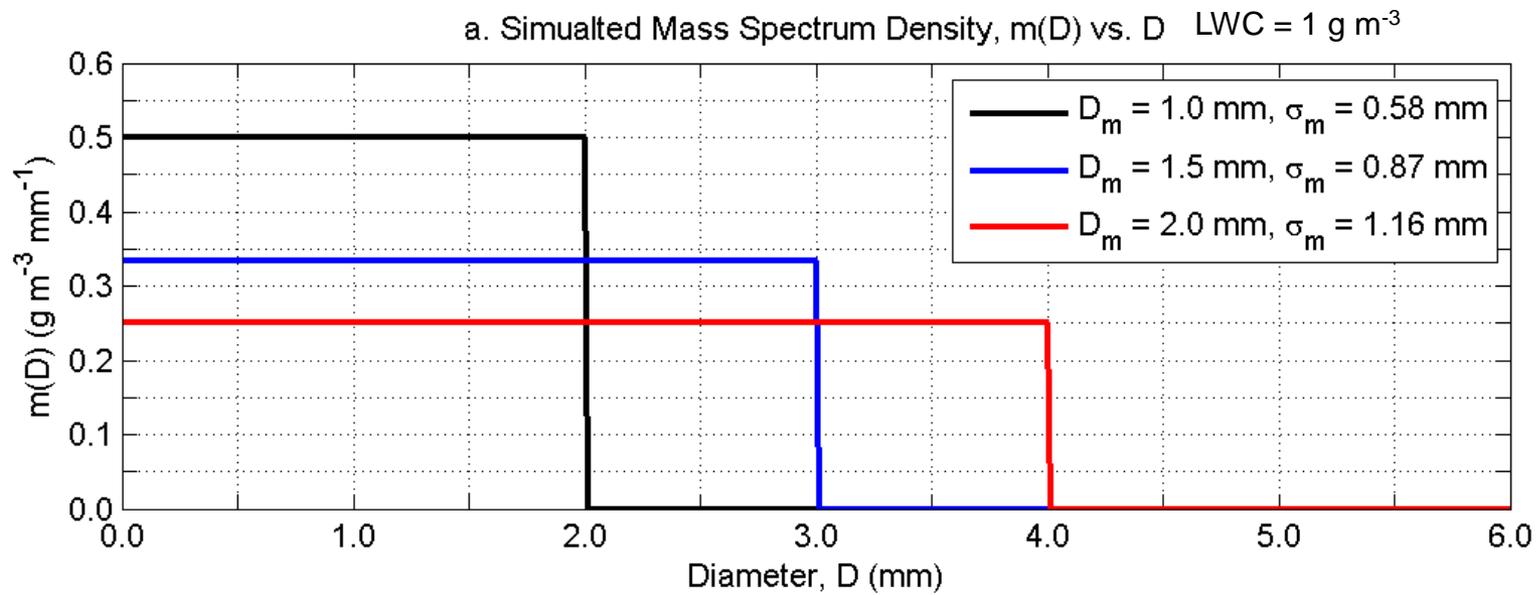
## Mass Spectrum Variance

$$\sigma_m^2 = \frac{\sum_{D_{\min}}^{D_{\max}} (D - D_m)^2 w(D) dD}{\sum_{D_{\min}}^{D_{\max}} w(D) dD}$$



# Top-Hat Mass Spectra

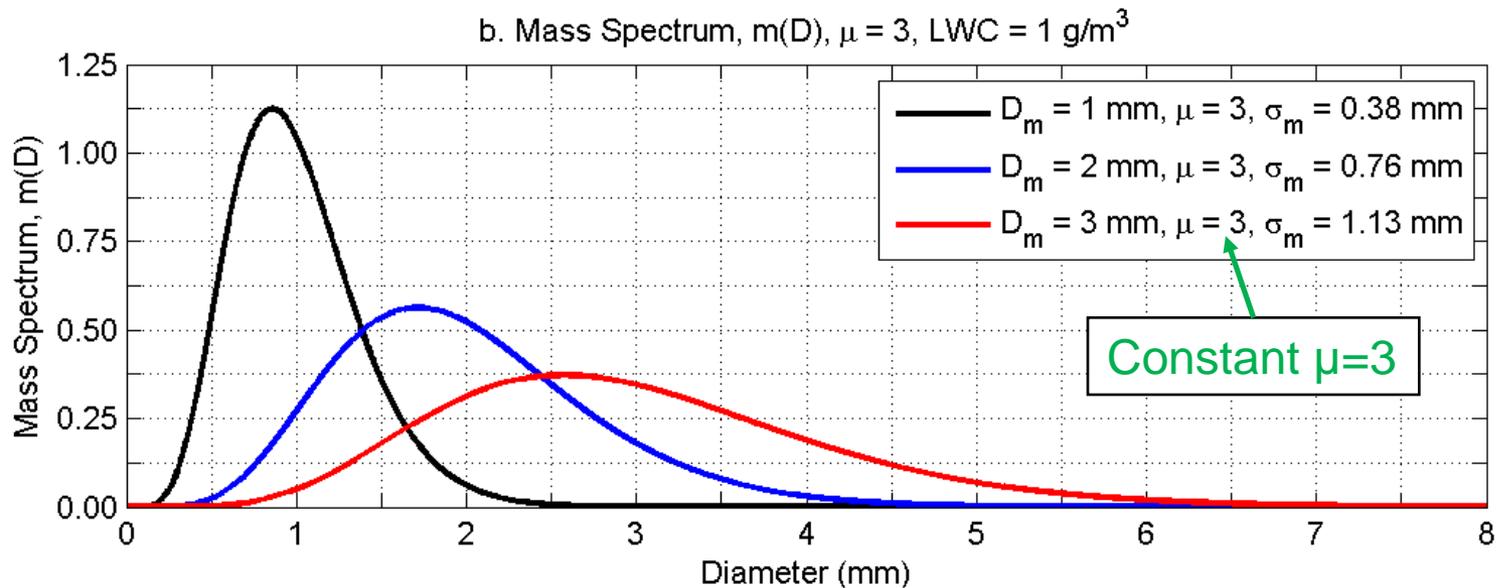
Raindrop spectra are 'single sided'  
(Can't have negative raindrop diameters)



Spectrum breadth increases as mean Diameter increases

# Simulated Gamma DSD

Raindrop spectra are 'single sided'  
(Can't have negative raindrop diameters)



Spectrum breadth increases as mean Diameter increases

# Disdrometer Observations (2DVD)

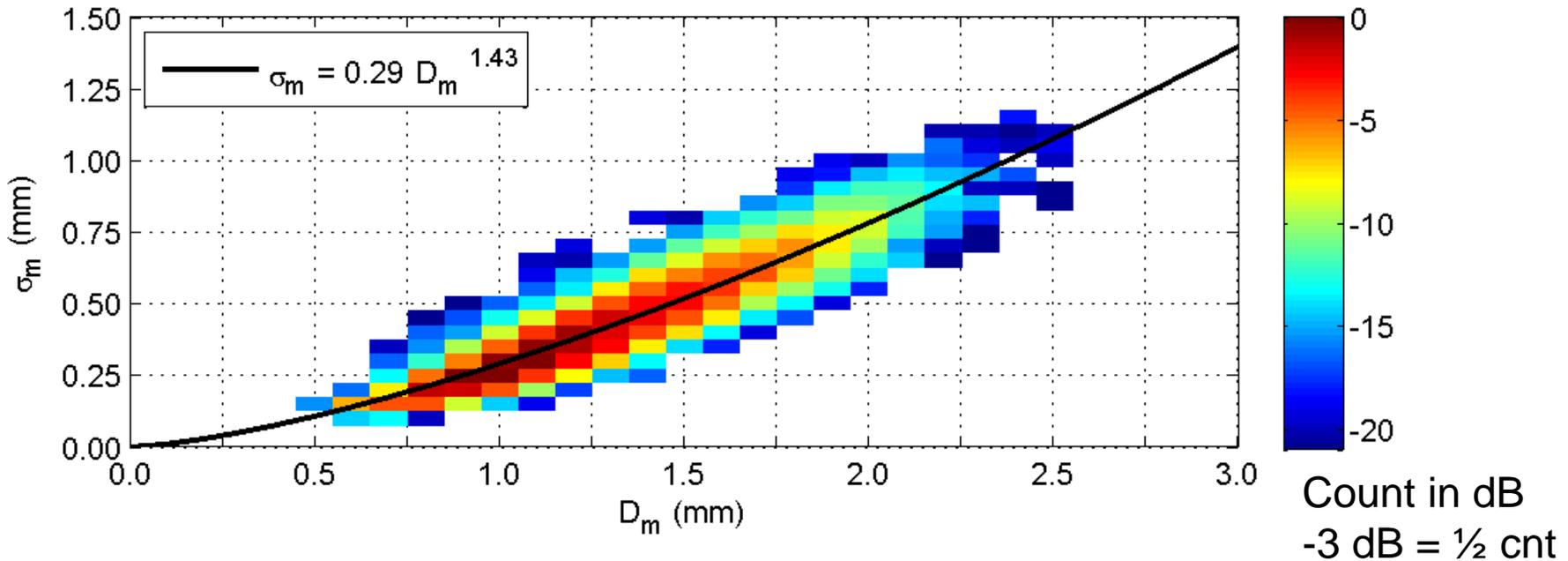
## Frequency of Occurrence

- Observed  $\sigma_m$  &  $D_m$
- No assumed DSD Shape
- Count is in dB
  - pixel with most counts = 0 dB
  - each -3 dB is half as many counts

Huntsville: 20,954 samples

$$\sigma_m = 0.29 D_m^{1.43}$$

a. Huntsville  $\sigma_m$  vs.  $D_m$



# Disdrometer Observations (2DVD)

If we **assume a gamma shape DSD**, there is a relationship between  $\sigma_m - D_m - \mu$   
(Assume the  $D_{max} = \infty$ ) (Ulbrich 1983)

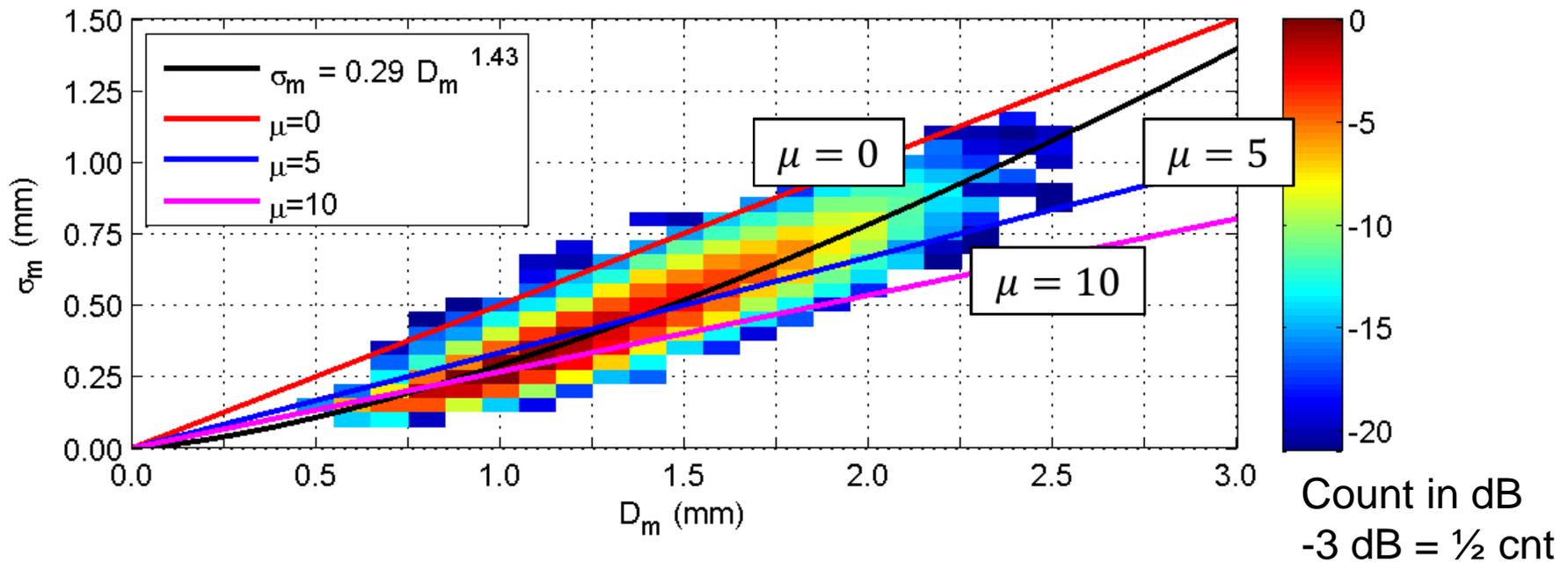
1. Can estimate  $\sigma_m$  from  $D_m$  and  $\mu$

$$\sigma_m^2 = \frac{D_m^2}{\mu + 4}$$

2. Can estimate  $\mu$  from  $D_m$  and  $\sigma_m$

$$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$

a. Huntsville  $\sigma_m$  vs.  $D_m$



# Disdrometer Observations (2DVD)

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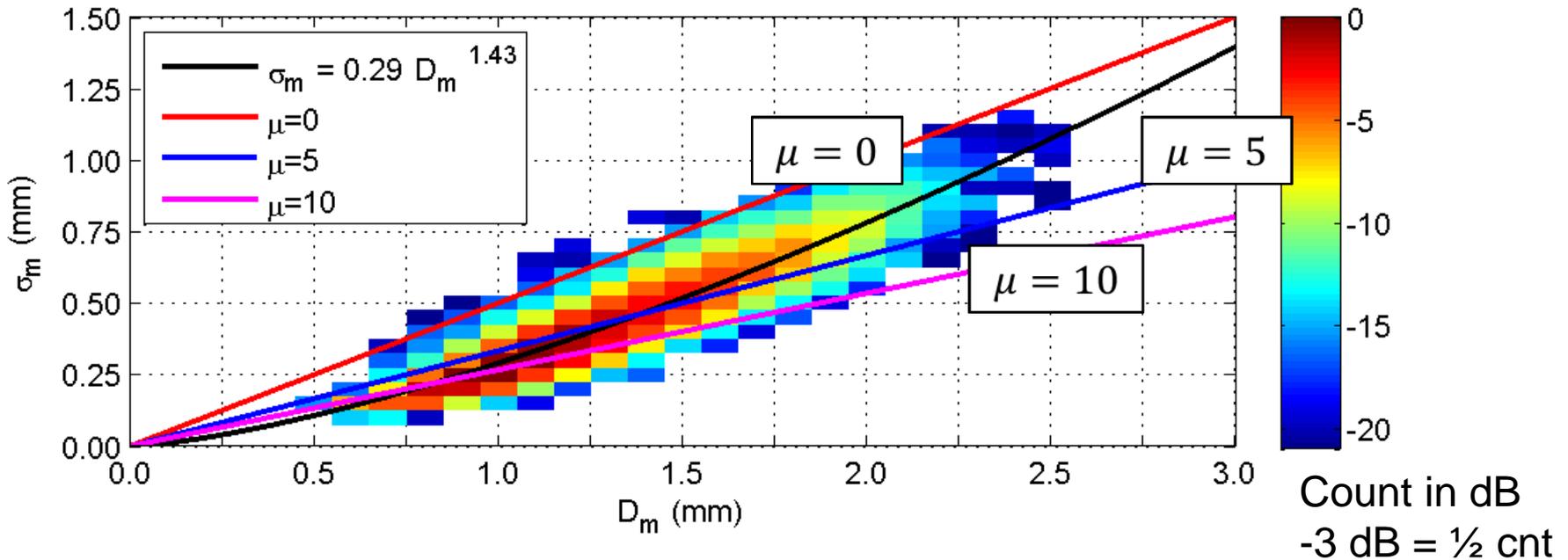
$$\sigma_m^2 = \frac{D_m^2}{\mu + 4}$$

2. Can estimate  $\mu$  from  $D_m$  and  $\sigma_m$

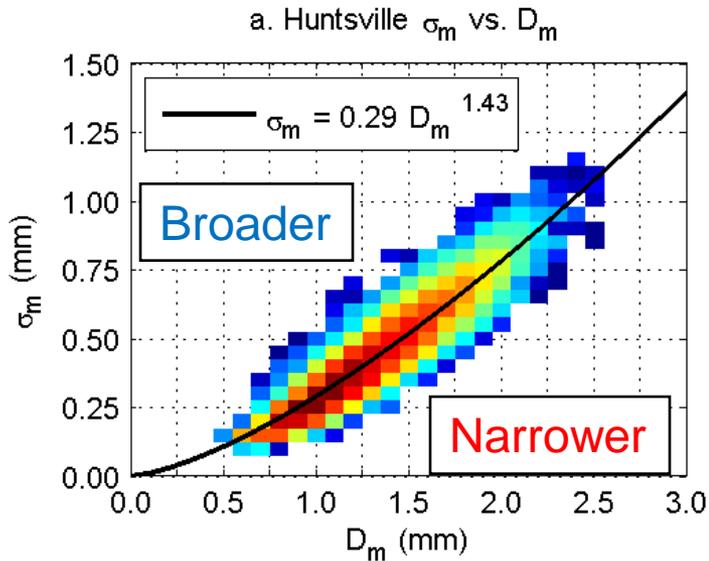
$$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$

Take-away message:  
 Family of gamma  
 functions can describe  
 the range of shapes that  
 are observed in  
 disdrometer mass spectra

a. Huntsville  $\sigma_m$  vs.  $D_m$



# Normalized Mass Spectrum STD



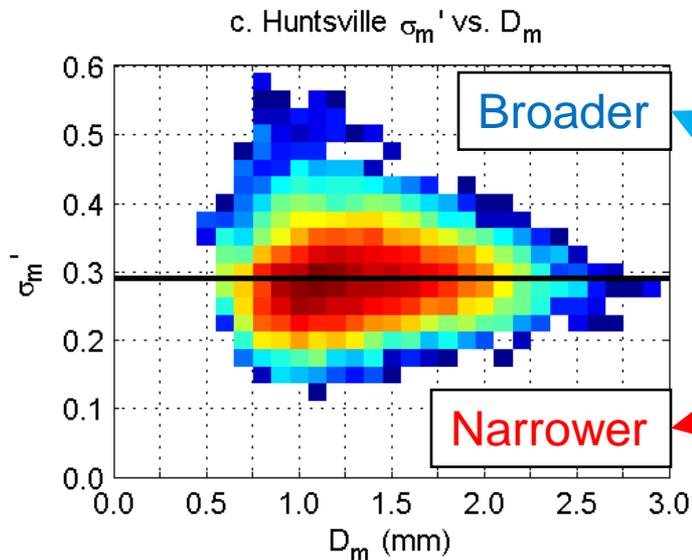
Power-law Relationships:

$$\sigma'_m = \frac{\sigma_m}{D_m^b}$$

$$\sigma_m = \sigma'_m D_m^b = \sigma'_m D_m^{1.43}$$

Uncorrelated:  $r^2(\sigma'_m, D_m) = 0$

*Haddad et al. (1996)*



Power-law represents expected value.

$$\sigma_m = \overline{\sigma'_m} D_m^b = 0.29 D_m^{1.43}$$

Broader Spectra:

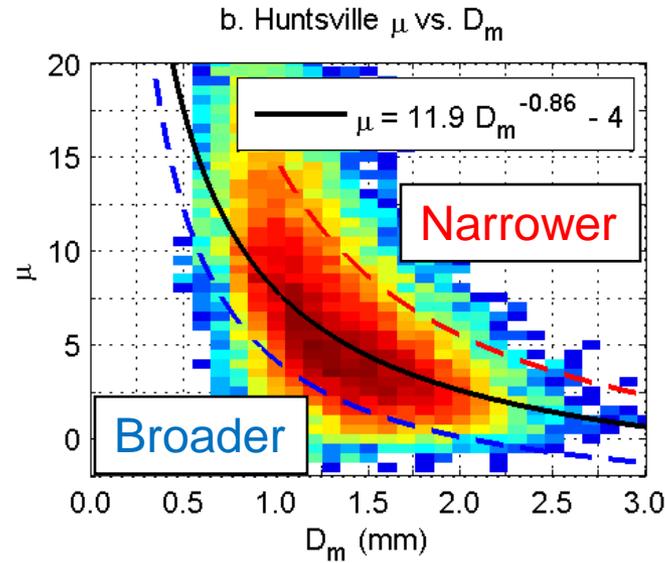
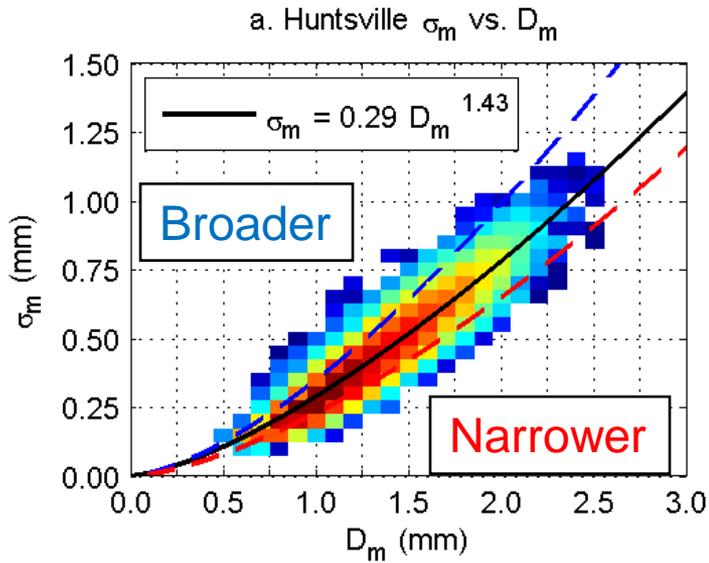
$$\sigma'_m > \overline{\sigma'_m}$$

Deviations have physical meaning

Narrower Spectra:

$$\sigma'_m < \overline{\sigma'_m}$$

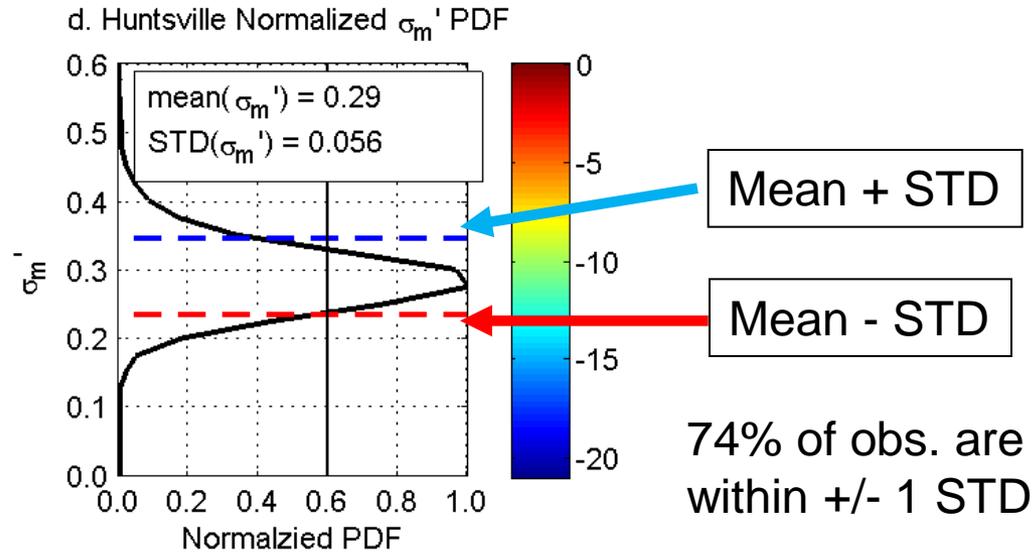
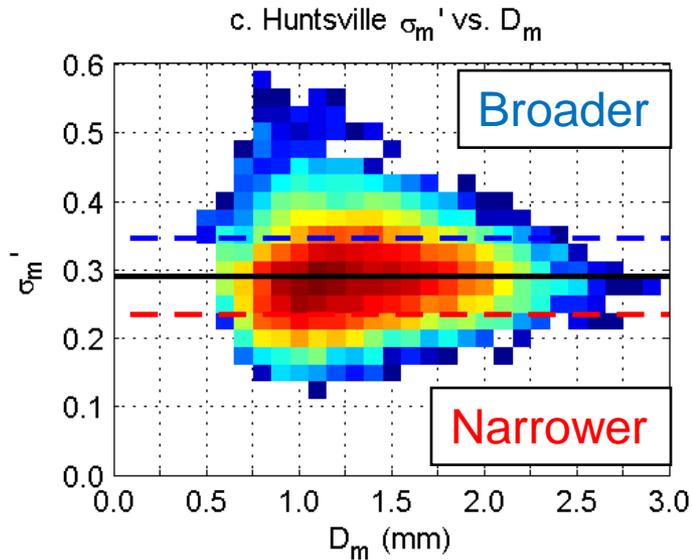
# Mapping into Gamma DSD Space



Assume:  
gamma DSD

$$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$

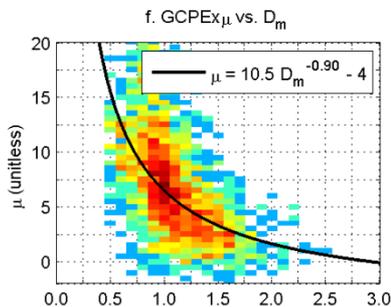
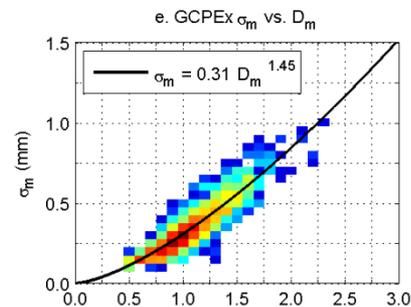
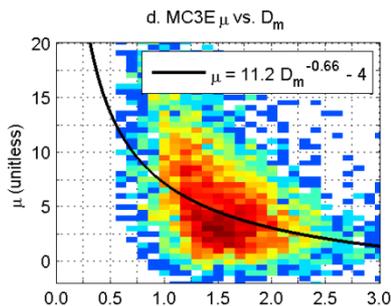
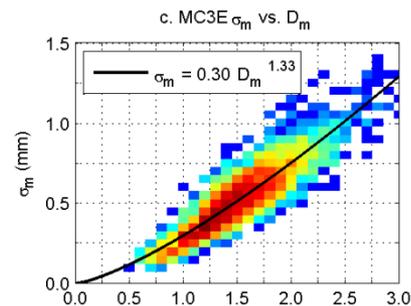
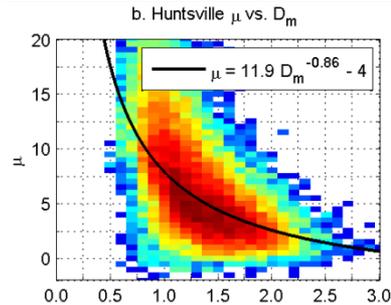
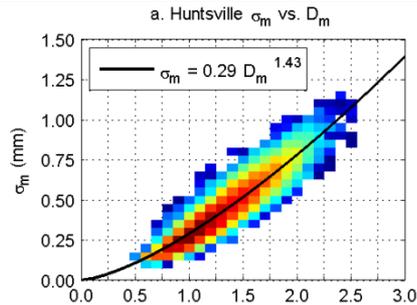
$$\sigma_m = \sigma'_m D_m^{1.43}$$



# Multiple Sites: Similar Power-laws

Observed  $\sigma_m$  vs.  $D_m$

Calculated  $\mu$  vs.  $D_m$



Huntsville: 20,954 samples

$$\sigma_m = 0.29 D_m^{1.43}$$

MC3E: 5,175 samples

$$\sigma_m = 0.30 D_m^{1.33}$$

GCPEX: 2,218 samples  
(Finland)

$$\sigma_m = 0.31 D_m^{1.45}$$

Ensemble: 27,347 samples

$$\sigma_m = 0.29 D_m^{1.42}$$

# Concluding Remarks

## Analyze ARM Data to evaluate model parameterizations

- What  $\sigma_m - D_m$  distributions are in cloud models?
- Can  $\sigma_m - D_m$  power-law be a model parameterization?

