# The Characterization of Hydrometeor Gamma Size Distributions as Volumes in $N_0/\mu/\lambda$ Phase Space: Implications for Microphysical Process Modeling

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## 1. Motivation

- Size distributions (SDs) parameterized in model schemes as gamma functions $N(D) = N_0 (D/D_0)^{\mu} e^{-\lambda D}$ where $D_0$ is a constant, assumed to be 1 cm

*N(D)* # distribution; D dimension; $N_0$ $\mu$ $\lambda$ fit parameters

- $N_0/\lambda/\mu$ determined from model-predicted moments or assumptions based on SDs measured in-situ
- In order to ultimately determine how in-situ $N_0/\lambda/\mu$ vary with meteorology, we need to investigate:

1. How accurately $N_0/\lambda/\mu$ can be determined from in-situ data?
2. How uncertainties in $N_0/\lambda/\mu$ affect model process rates?

## 2. Fit Techniques

- **Data:** Two-D Stereo, Cloud & Precipitation Probe (2DS/C/P) from 2008 Indirect & Semi-Direct Aerosol Campaign (ISDAC) and Cloud & Precipitation Imaging Probe (CIP/PIP) from NASA African Monsoon Multi-disciplinary Analyses (NAMMA) define 10-s SDs
- **Moments:** 1) N$^{th}$ moment of observed SD at observed sizes $D_i$ is $M^*_n = \sum_i N^*(D_i) D_i^{\,n} \Delta D_i$ ; 2) N$^{th}$ moment of SD between minimum ($D_{min}$) & maximum ($D_{max}$) D is $M_n = \int_{D_{min}}^{D_{max}} N(D) D^n dD$
- **SDs fit to gamma distribution using 5 techniques:**

1) Incomplete Gamma Fit (IGF, McFarquhar et al. 2012) minimizing $\chi^2$ between observed & fit moments accounting for data not covering all sizes $\chi_{IGF}{}^2 = \sum_{n=1,2,6} \left(\frac{M_n - M^*_n}{\sqrt{M_n M^*_n}}\right)^2$
2) discrete gamma fit, like IGF, except $M_n$ computed at same bins as $M^*_n$
3) Standard gamma fit minimizing $\chi^2$ between fit $N(D_i)$ & observed $N^*(D_i)$ $\chi_{sg}{}^2 = \sum_i (N(D_i) - N^*(D_i))^2$
4) normalized gamma fit where $\chi^2$ in 3) weighted by inverse of $N^*(D_i)$ $\chi_{ng}{}^2 = \sum_i \left(\frac{(N(D_i) - N^*(D_i))}{N^*(D_i)}\right)^2$
5) Solution to $(1-F)\mu^4 + (8-18F)\mu^3 + (24-119F)\mu^2 + (32-342F)\mu + (16-360F) = 0$ where $F = M^{*}_2{}^5 / M^*_6 M^{*}_1{}^4$ (Heymsfield et al. 2002)

- **Comparison of Fit Techniques**

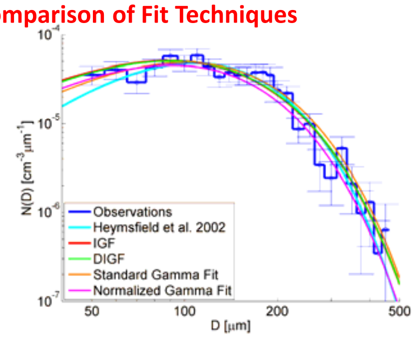



*Fig. 1: SD derived from 2DS, 2DC & 2DP data for 10-s period during ISDAC with best fit for all 5 techniques.*

- Visually 5 fits appear similar
- But, there is huge range in $N_0/\lambda/\mu$

IGF: $N_0 = 4.9\times10^{0}$ cm$^{-3}$ µm$^{-1}$, $\mu = 2.0$; $\lambda = 2.2\times10^{-1}$ µm$^{-1}$

H02: $N_0 = 1.1\times10^{-3}$ cm$^{-3}$ µm$^{-1}$, $\mu = 3.1$; $\lambda = 2.8\times10^{-2}$ µm$^{-1}$

→ Can't represent SD by single $N_0/\lambda/\mu$; need ranges

→ Develop volumes which give all $N_0/\lambda/\mu$ with $\chi^2 < \chi_{min}{}^2 + \Delta\chi^2$, which are **equally realizable solutions**

→ $\Delta\chi^2$ determined from uncertainties in SDs & Jakobian of $\chi^2$ derived from fit

## 3. Volumes in $N_0/\mu/\lambda$ Phase Space

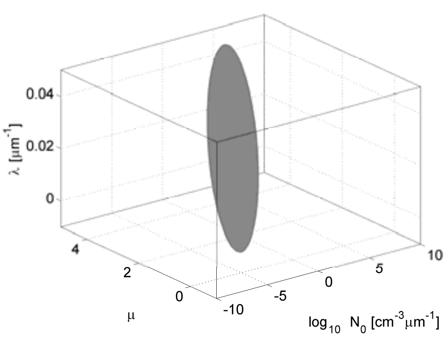



Fig. 2: Volume of equally realizable solutions in ($N_0$, $\mu$, $\lambda$) phase space text for SD depicted in Fig. 1.

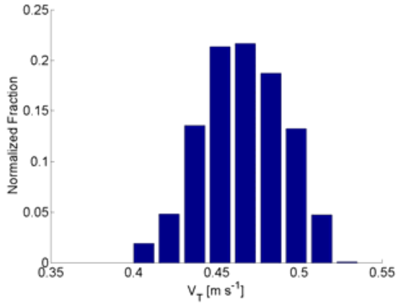



Fig. 4: Normalized frequency distribution of mass-weighted fall speed, $V_q$, using $N_0/\lambda/\mu$ randomly selected from surface in Fig. 2; there is ~10% variation in $V_q$ depending on $N_0/\lambda/\mu$ selected

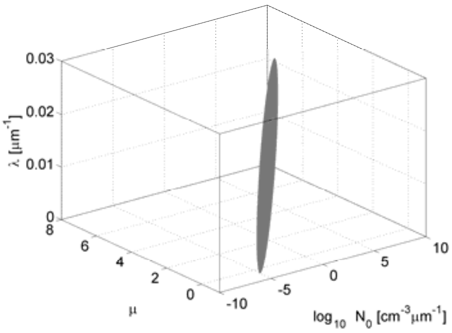



Fig. 3: As in Fig. 2; except for SD measured during NAMMA

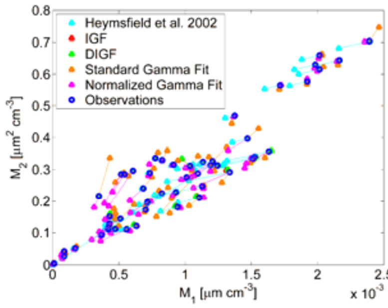
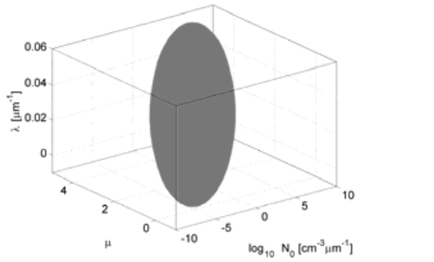



Fig. 5: Comparison of observed & simulated moments for NAMMA SDs; IGF best matches observed moments

## 4. Characterizing Families of SDs



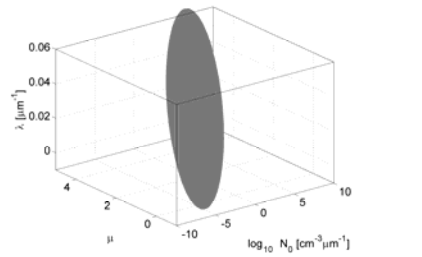




Fig. 6 (left) and 7 (right): Ellipsoids characterizing ISDAC (left) and NAMMA (right) SDs include all points included within 1% of individual SD ellipses: eigenvectors & eigenvalues of Hessian matrix allow use of these volumes in numerical models. Need to determine how ellipsoids vary with cloud and environmental parameters. Reasons for differences between ISDAC and NAMMA volumes currently being investigated.

## 5. Conclusions

1. IGF technique accounting for fact in-situ data does not cover all particle sizes gives better estimate of bulk observed moments than other techniques.
2. Volume of $N_0/\lambda/\mu$ values, determined from uncertainty of observed SDs, characterizes fit to each SD and gives mutual dependence of fit parameters.
3. Uncertainty of about 10% in $V_q$ seen from uncertainty in of $N_0/\lambda/\mu$.
4. Currently using these techniques to investigate how SD parameters vary with cloud and environmental parameters.

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