More Analysis of Cirrus Cloud Particle Size Distributions Measured During Sparticus

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Data provided by Paul Lawson of SPEC
Motivation

- Remote sensing of cirrus particle size distributions (PSD’s)
- Maximize the posterior distribution of possible PSD params given remote observations, as given by Bayes’ Rule:

\[
p(\vec{x} | \vec{y}) \propto L(\vec{y} | \vec{x})p(\vec{x})
\]

- Measurement and pram vectors

\[
\vec{x} = \begin{bmatrix}
p_1 \\
... \\
p_n
\end{bmatrix} \quad \vec{y} = \begin{bmatrix}
Z \\
... \\
Tb
\end{bmatrix}
\]
Focus on the Prior Distribution:

\[ p(\bar{x}) \]

- Often assumed to be Gaussian (e.g. Rogers, 2000)

\[
p(\bar{x}) = \frac{1}{(2\pi)^{n/2}|S_a|^{1/2}} \exp \left[ -\frac{1}{2} (\bar{x} - \bar{x}_a)^T S^{-1}_a (\bar{x} - \bar{x}_a) \right]
\]

- Covariance generally assumed to be diagonal

\[
S_a = \begin{bmatrix}
\sigma_1^2 & 0 & 0 \\
0 & \sigma_2^2 & 0 \\
0 & 0 & \sigma_3^2
\end{bmatrix}
\]
PSD Fits

- Data from 81 of Sparticus’ flight legs
- 2-DS distributions fit with

\[ n(D) = N_0 (D/D_0)^\alpha \exp(-D/D_0); \quad \mathbf{x} = [N_0 \quad D_0 \quad \alpha]^T \]

- and

\[ n(D) = n_l(D) + n_s(D); \quad \tilde{\mathbf{x}} = \begin{bmatrix} N_{0l} & D_{0l} & \alpha_l & N_{0s} & D_{0s} & \alpha_s \end{bmatrix}^T \]

- ~25,000 fits
CloudSat Overpass of Sparticus, 2/3/10

CloudSat dBZ and Lidar Mask 2010034_20059

Natural log #/liter/micron

Latitude

Height (km)
Unimodal Distribution Fit with Maximum Likelihood Algorithm

$\log_{10}(N_0)$ for all flights, degenerates removed

$\log_{10}(D_0)$ for all flights, degenerates removed

alpha for all flights, degenerates removed
Bimodal Distribution Fit with Method of Moments/Excess Mass Algorithm
## Covariance Analysis for Both Fits

\[
S_{uni} = \text{cov} \begin{bmatrix}
\log_{10}(N_0) \\
\log_{10}(D_0) \\
\alpha
\end{bmatrix} = 
\begin{bmatrix}
12.18 & 0.22 & -13.35 \\
0.022 & 0.14 & -0.61 \\
-13.35 & -0.61 & 16.43
\end{bmatrix}
\]

\[
S_{bi} = \text{cov} \begin{bmatrix}
\log_{10}(N_l) \\
\log_{10}(D_l) \\
\alpha_l \\
\log_{10}(N_s) \\
\log_{10}(D_s) \\
\alpha_s
\end{bmatrix} = 
\begin{bmatrix}
0.89 & -0.22 & -0.14 & 0.33 & -0.21 & 0.38 \\
-0.22 & 0.12 & -0.03 & -0.04 & 0.12 & -0.30 \\
-0.14 & -0.03 & 0.42 & -0.01 & -0.002 & 0.02 \\
0.33 & -0.04 & -0.01 & 1.34 & -0.10 & -1.06 \\
-0.21 & 0.12 & -0.002 & -0.10 & 0.17 & -0.43 \\
0.38 & -0.30 & 0.02 & -1.06 & -0.43 & 2.86
\end{bmatrix}
\]
Whether the Unimodal or Bimodal Fit is More Likely Correct Tested Using a Likelihood Ratio Test
Common Examples of Bimodal and Unimodal Fits, Correctly Flagged
Some Counter-Examples
Scattered Moments from the Two Fit Distributions

- **Zeroth Moment**
- **First Moment**
- **Second Moment**
- **Third Moment**
- **Sixth Moment**

Legend:
- Red diamond: Unimodal, Maximum Likelihood
- Blue circle: Bimodal, Method of Moments
Scattered Moments from Either Distribution

- **Zeroth Moment**
- **First Moment**
- **Second Moment**
- **Third Moment**
- **Sixth Moment**
Bimodality and Temperature

Histogram of Temperature

Two-sample Kolmogorov-Smirnov test confirms that the two temperature histograms were drawn from different distributions.
Summary and Conclusion

- Fit with a uni- or bimodal distribution function, the prior distribution of PSD parameters is not Gaussian and does not have a diagonal covariance matrix.
- A likelihood ratio was used to determine whether fit was more appropriate.
- The bimodal model is not always better than the unimodal model, even in cases that exhibit bimodality.
- If a bimodal distribution is to be used, a more sophisticated model needs to be developed.
- Cirrus PSD retrievals can be informed by meteorological situation.