

# Quantifying aerosol mixing state with entropy measures

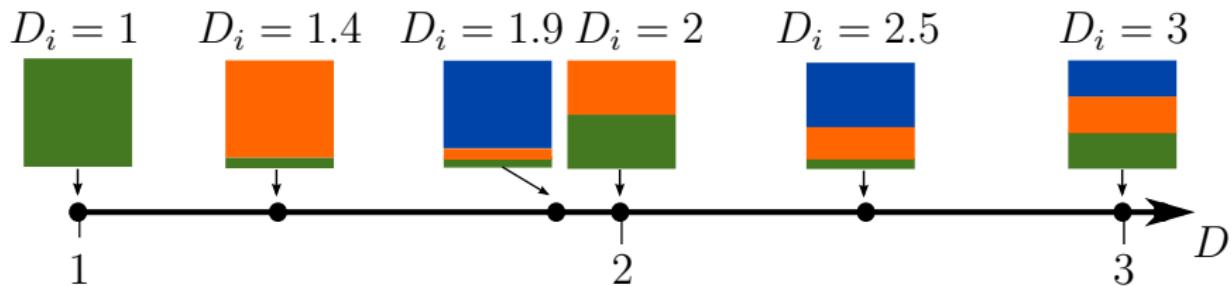
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March 21st, 2013

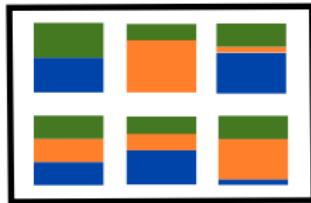
# Single-particle diversity

- $p_i^a$  = mass fraction of species  $a$  in particle  $i$
- mixing entropy of  $i$ th particle:  $H_i = -\sum_a p_i^a \ln p_i^a$
- diversity of  $i$ th particle:  $D_i = e^{H_i}$  (units of *effective species*)



- Concepts independently discovered in information theory, statistical mechanics, ecology, genetics, finance, etc.

# Population-level diversities



$$\underbrace{D_\alpha}_{\text{per-particle diversity}} \times \underbrace{D_\beta}_{\text{inter-particle diversity}} = \underbrace{D_\gamma}_{\text{bulk population diversity}}$$

- average particle diversity:  $D_\alpha = e^{H_\alpha}$ ,  $H_\alpha = \sum_i p_i H_i$
- bulk population diversity:  $D_\gamma = e^{H_\gamma}$ ,  $H_\gamma = - \sum_a p^a \ln p^a$

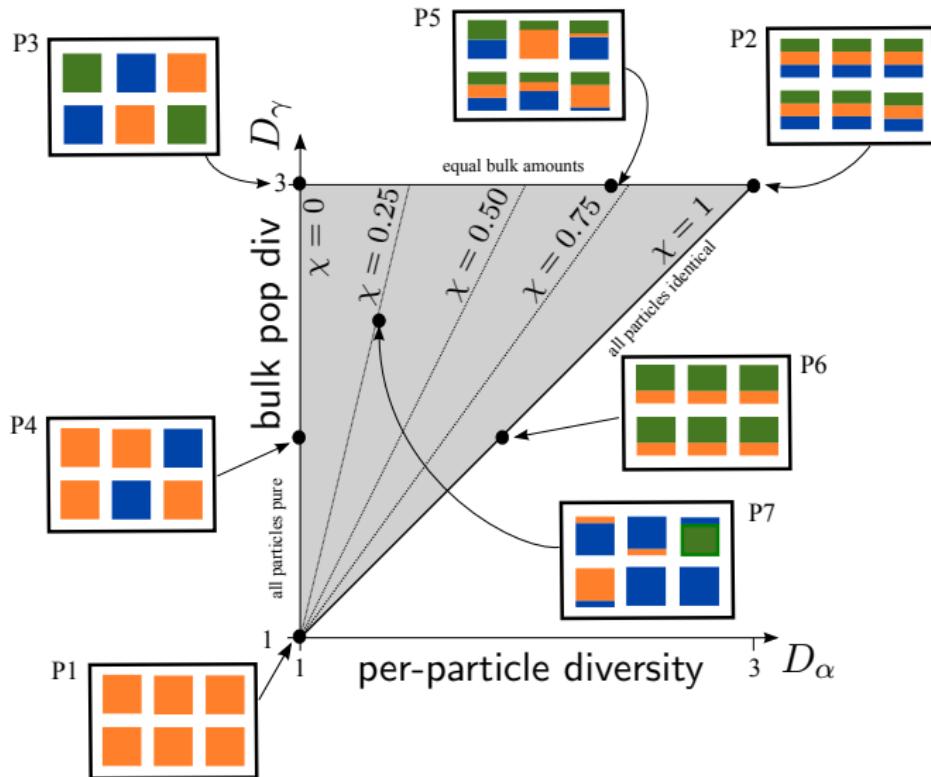
where:

- $p_i$  = mass fraction of particle  $i$  in the population
- $p^a$  = mass fraction of species  $a$  in the population

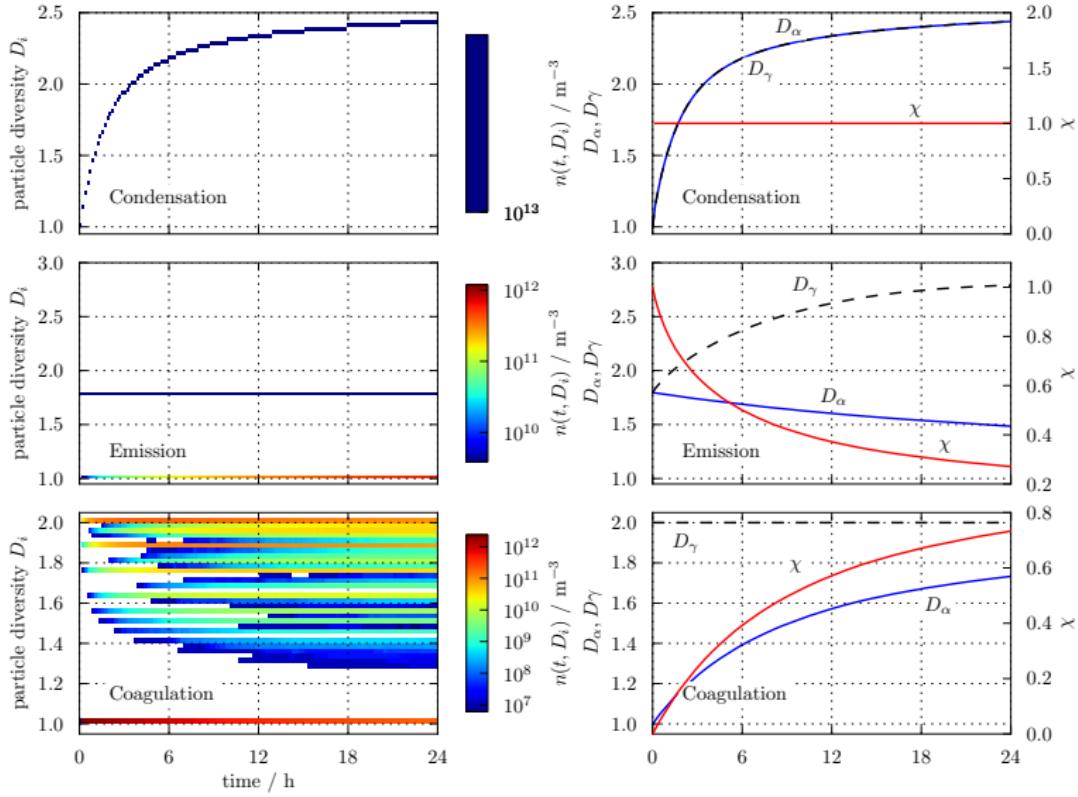
then:

- inter-particle diversity:  $D_\beta = \frac{D_\gamma}{D_\alpha}$
- mixing state index:  $\chi = \frac{D_\alpha - 1}{D_\gamma - 1}$

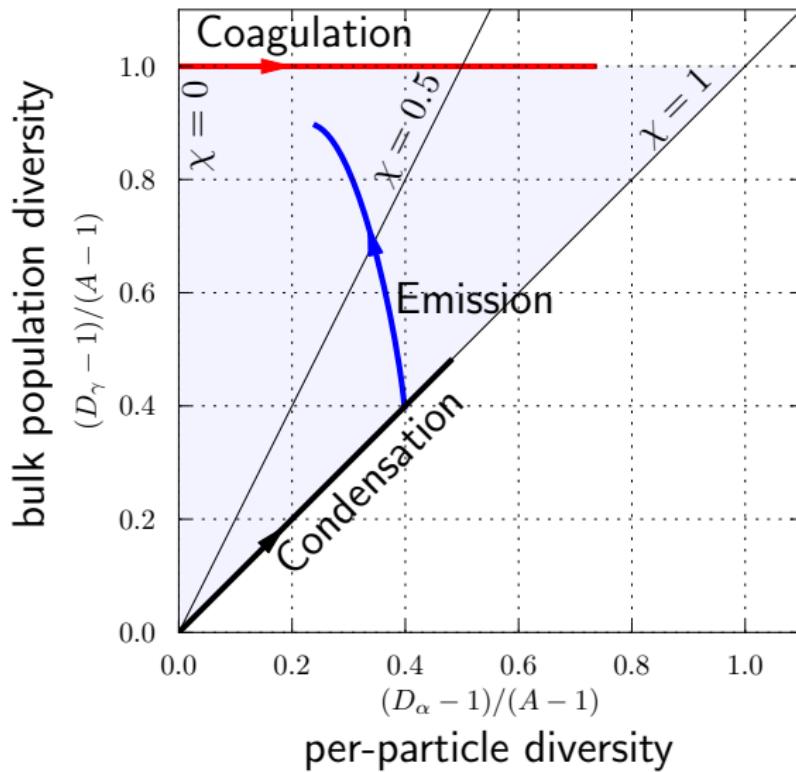
# Mixing state diagram



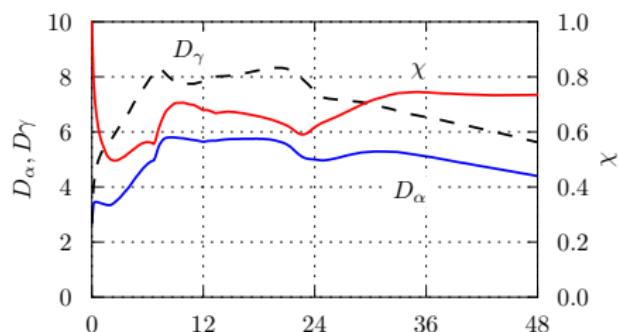
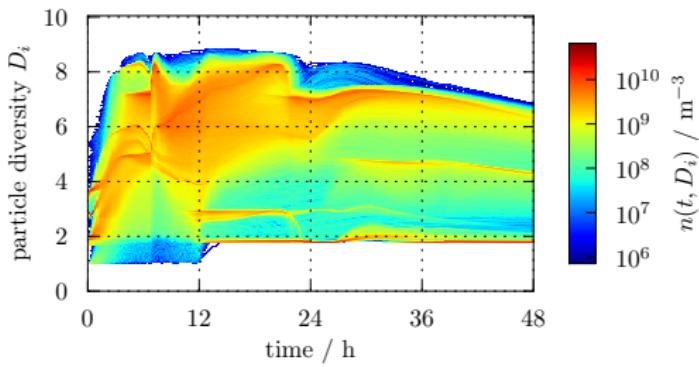
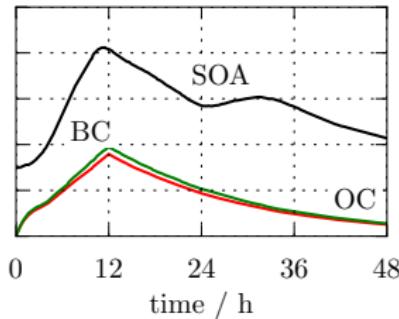
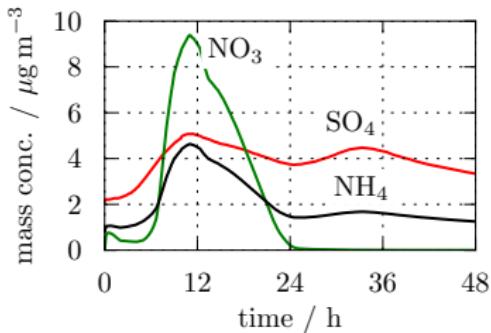
# Archetypal cases



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# Urban plume PartMC simulation



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