# Relationships between DSD Parameters Observed in MC3E Observations

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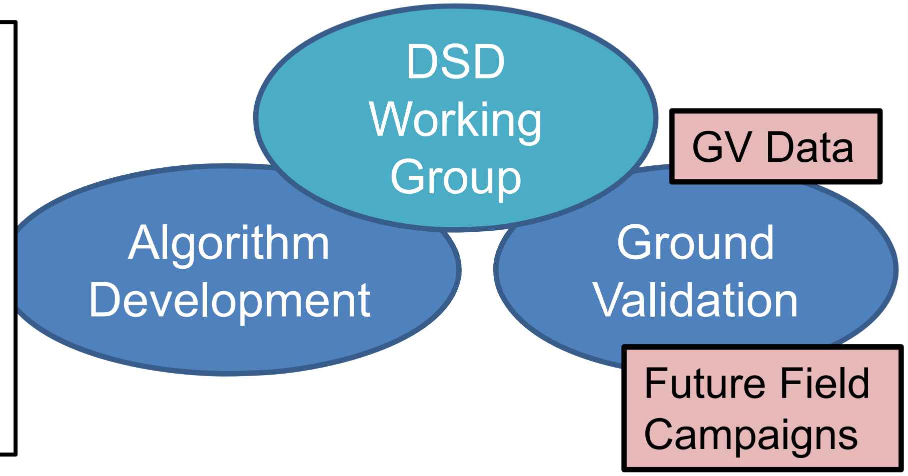
# NASA GPM DSD Working Group: Bridging Algorithms and Ground Validation (GV)

**General Objective:** Use Ground Validation (GV) data to investigate relationships between DSD parameters that support, or guide, the assumptions used in satellite retrieval algorithms.
**Rationale:** Relationships between DSD parameters, if found, can be used to constrain the unknowns in satellite algorithms.




With guidance from Algorithm Developers, we are using previously collected GV data (point, columnar, and spatial GV data sets) to address these objectives:

1. Develop physically based relationships between DSD parameters.
2. Develop a framework to incorporate GV findings into Algorithms.

Discussed today

3. Describe the vertical structure of DSD parameters.
4. Investigate snow size parameters and their correlations.

Future Work

*DSD Working Group Monthly Teleconference calls: 3rd Thursday @ 1 PM Eastern.*

Define Gamma shaped DSD, $N_w$, $D_m$, $\mu$:

$$N(D;N_w,D_m,\mu)=N_w f(\mu)\left(\frac{D}{D_m}\right)^{\mu}\exp\left(-\frac{(4+\mu)}{D_m}D\right)$$

Difficult to estimate μ and $D_m$ from individual N(D) spectra because μ and $D_m$ are correlated (Chandrasekar & Bringi 1989)

To avoid fitting artifacts, do not estimate gamma DSD parameters.
Find relationships between Mass Spectrum Parameters (no assumed DSD shape).

**Mass Spectrum**

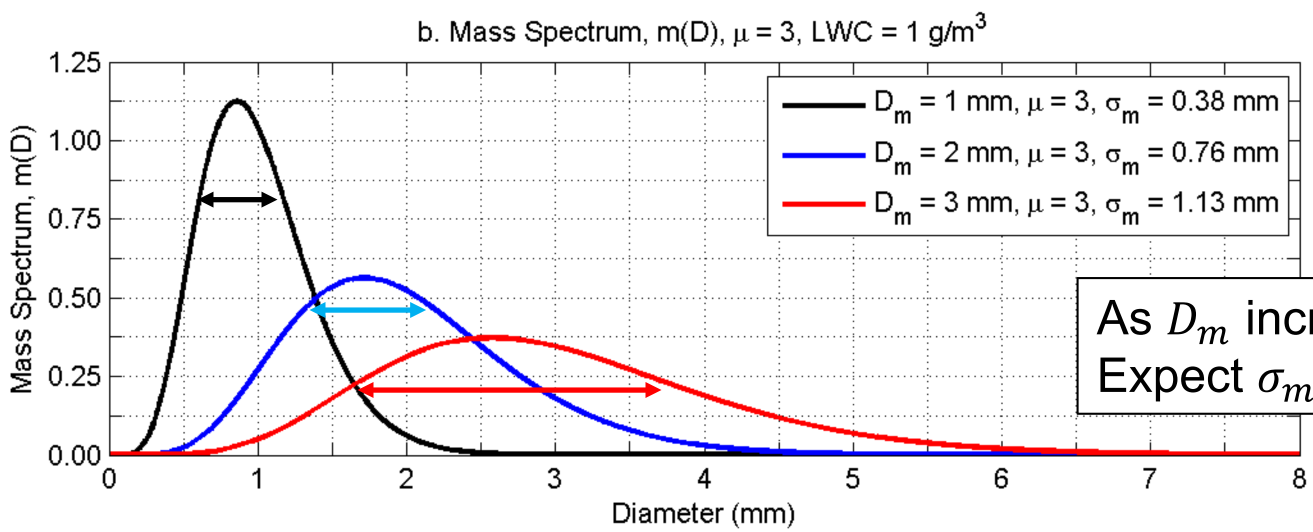
$$w(D)=\frac{\pi}{6\cdot 10^3}\rho_w N(D)D^3$$

**Mean Diameter**

$$D_m=\frac{\sum_{D_{\min}}^{D_{\max}} w(D)D\,dD}{\sum_{D_{\min}}^{D_{\max}} w(D)\,dD}$$

**Mass Spectrum Variance**

$$\sigma_m^2=\frac{\sum_{D_{\min}}^{D_{\max}} (D-D_m)^2 w(D)\,dD}{\sum_{D_{\min}}^{D_{\max}} w(D)\,dD}$$

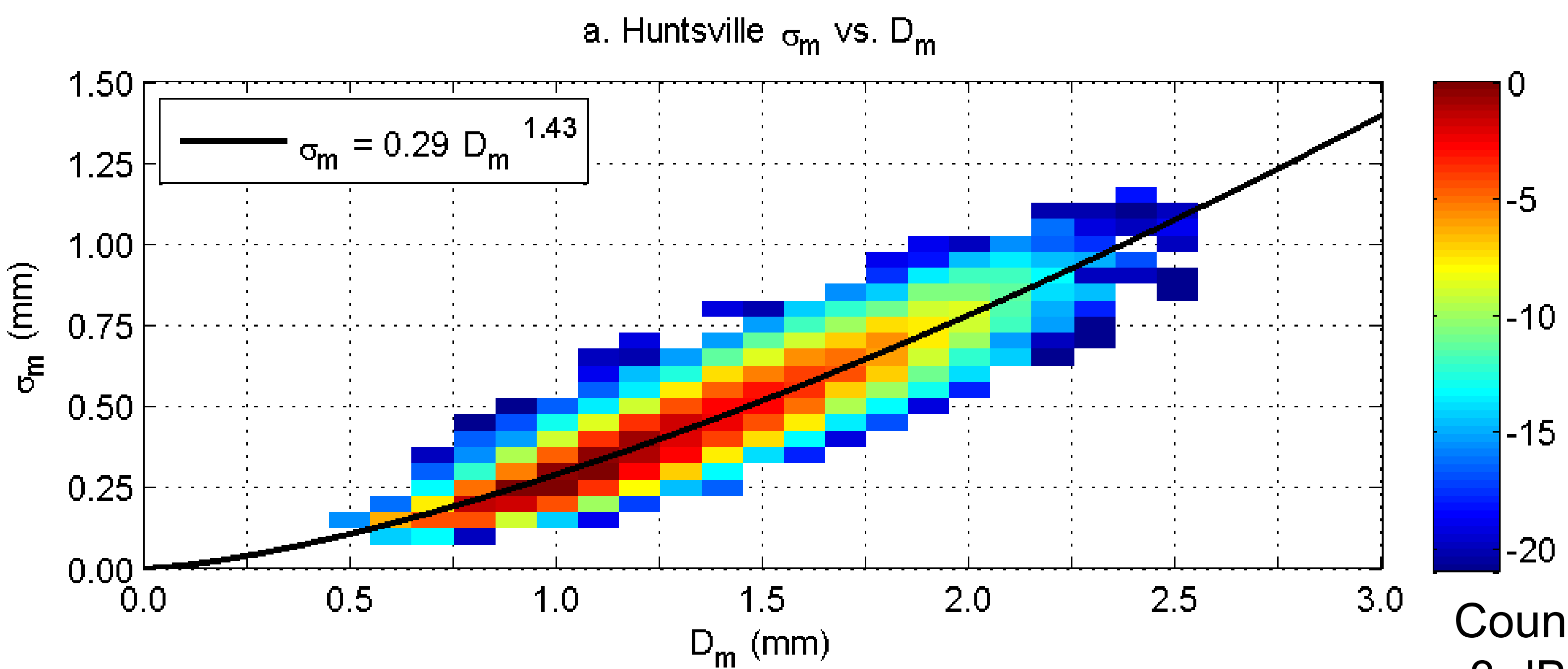



As $D_m$ increases, Expect $\sigma_m$ to increase

# Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples

**Frequency of Occurrence**

- Observed $\sigma_m$ & $D_m$
- No assumed DSD Shape
- Count is in dB
  - pixel with most counts = 0 dB
  - each -3 dB is half as many counts




Count in dB
-3 dB = ½ cnt

# Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples

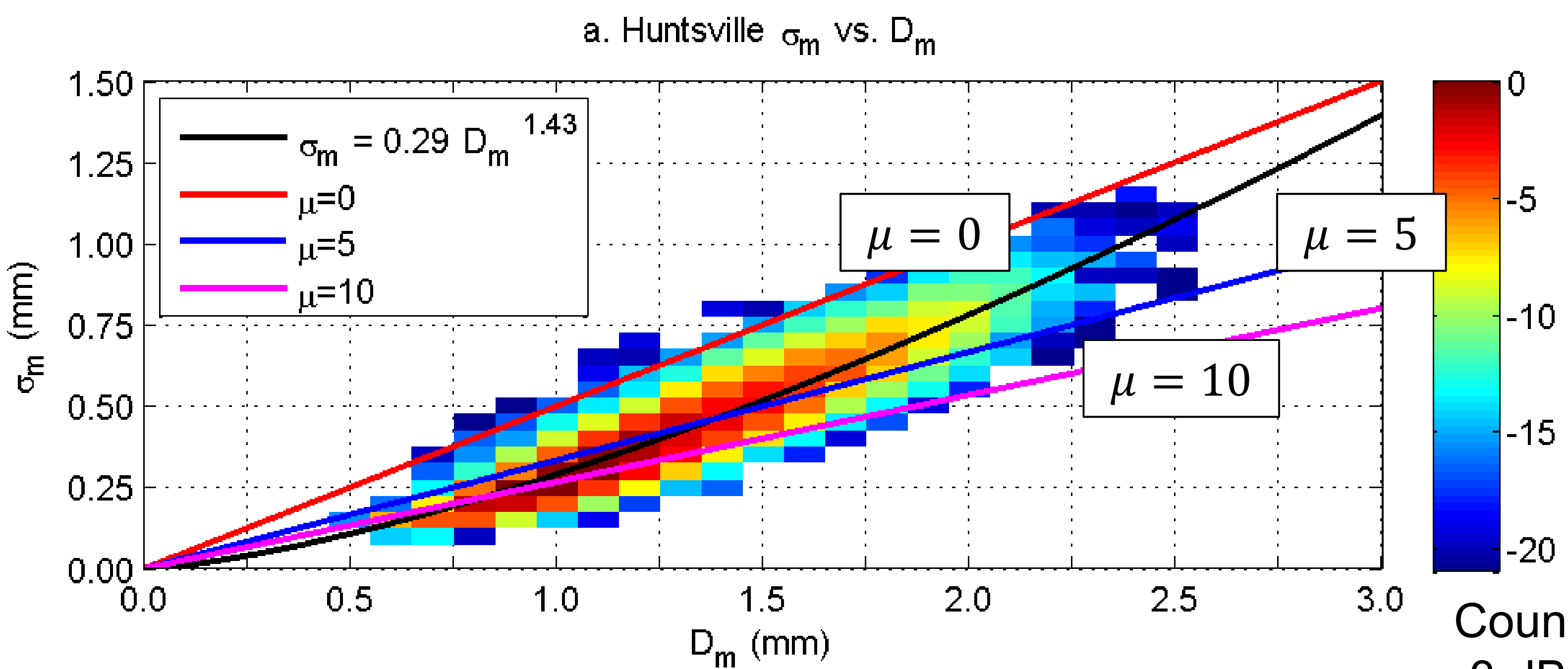
If we assume a gamma shape DSD, there is a relationship between $\sigma_m - D_m - \mu$
(Assume the $D_{max} = \infty$)

1. Can estimate $\sigma_m$ from $D_m$ and $\mu$

$$\sigma_m^2 = \frac{D_m^2}{\mu + 4}$$

2. Can estimate $\mu$ from $D_m$ and $\sigma_m$

$$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$




Count in dB
-3 dB = ½ cnt

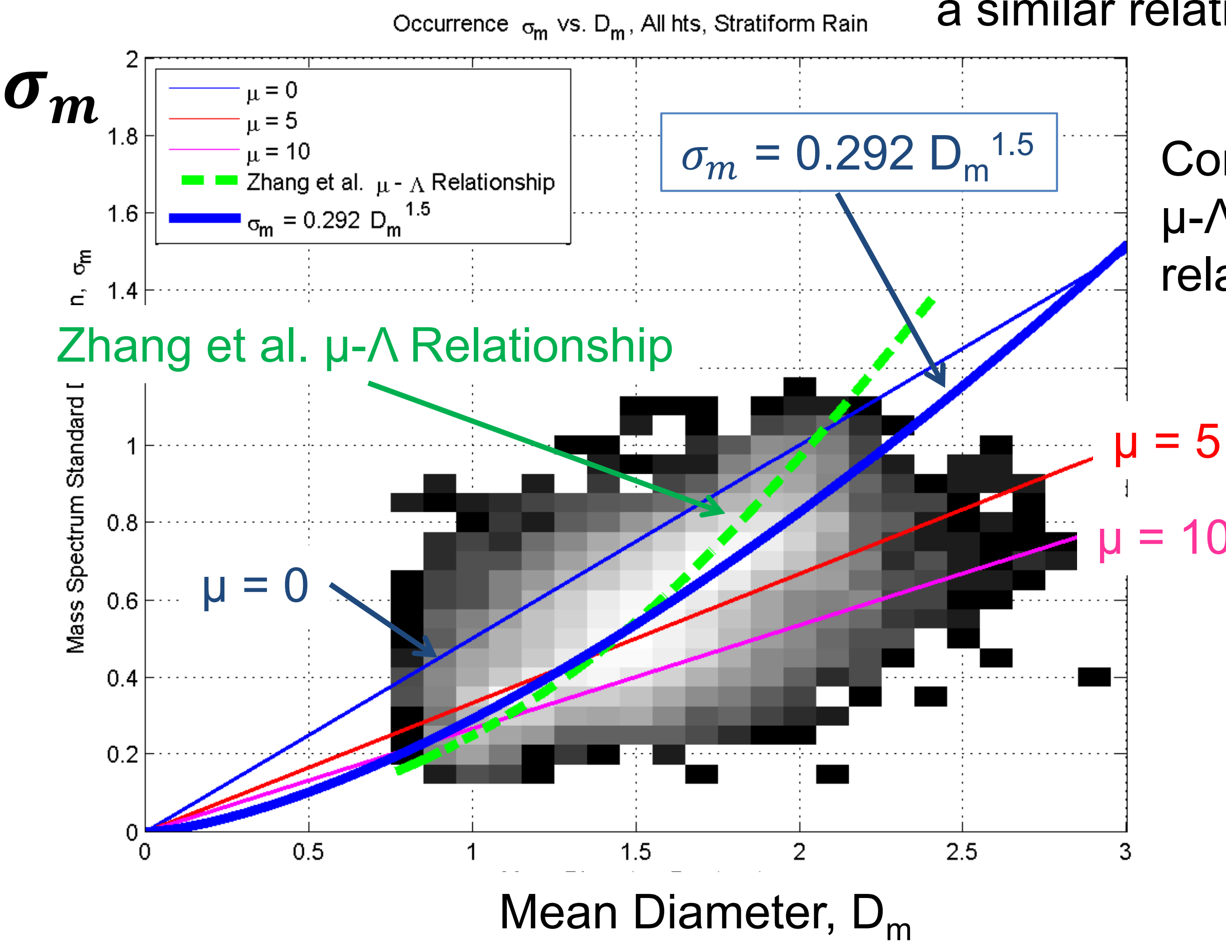
# Darwin Profiler Retrieved DSDs

## $\sigma_m$ vs. $D_m$ for all pixels

## Zhang et al. (2001) μ-Λ Relationship

$$\Lambda = 0.365\mu^2 + 0.735\mu + 1.935$$

Brandes et al. (2003) found a similar relationship



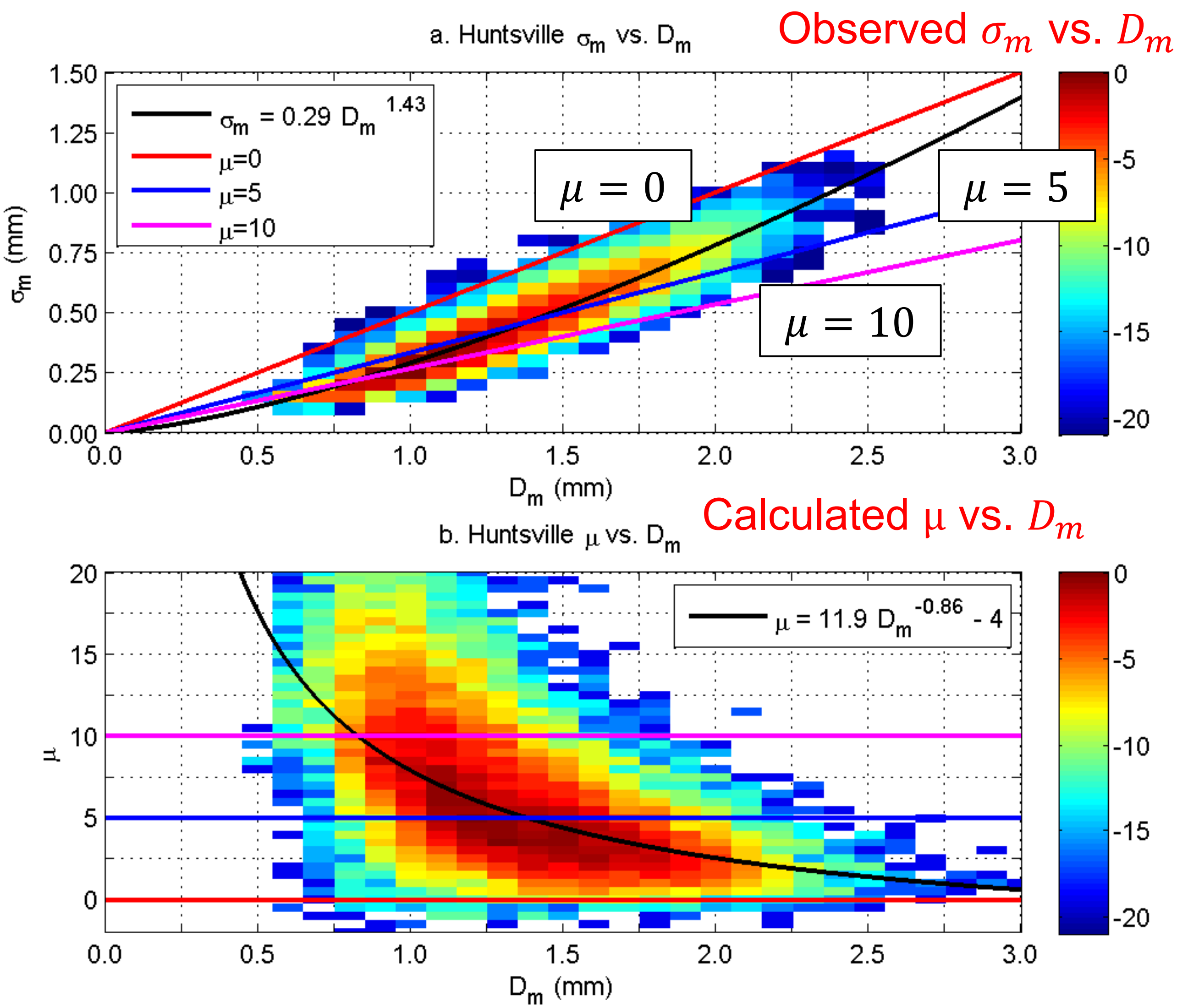

Convert Zhang et al. μ-Λ into $\sigma_m - D_m$ relationship using:

$$\Lambda = \frac{4+\mu}{D_m}$$

and

$$\frac{\sigma_m^2}{D_m^2} = \frac{1}{4+\mu}$$

# Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples




Estimate $\sigma_m$ from $D_m$ and $\mu$ using:

$$\sigma_m^2 = \frac{D_m^2}{\mu + 4}$$

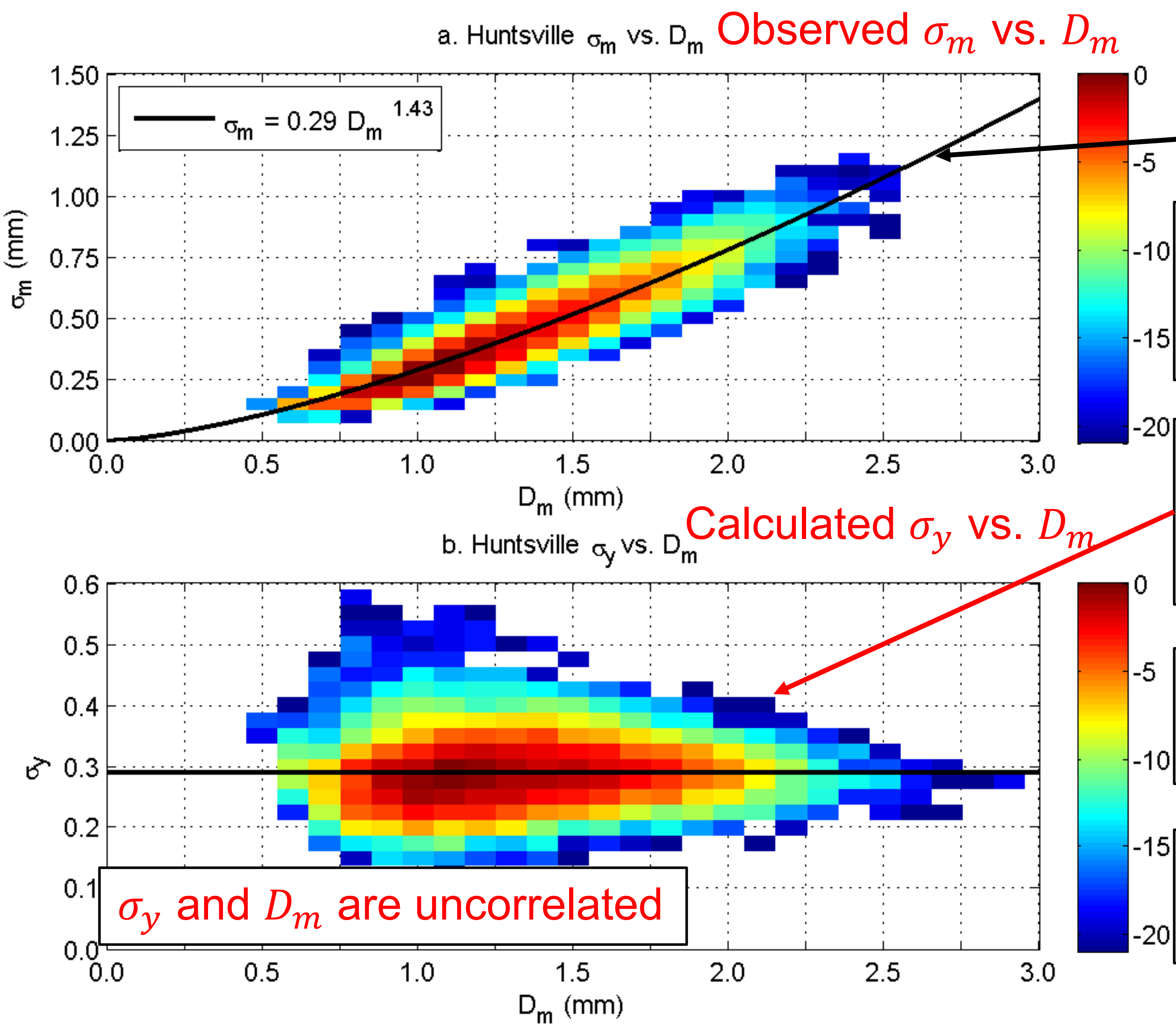
Estimate $\mu$ from $D_m$ and $\sigma_m$ using:

$$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$

For this dataset, $\mu$ Power-law is:

$$\mu = \frac{11.9}{D_m^{-0.86}} - 4$$

# Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples




How is the Power-law fit determined?

$$\sigma_m = a\ D_m^b$$

1. Define a new variable:

$$\sigma_y = \frac{\sigma_m}{D_m^b}$$ Observed

2. Adjust $b$, until $\sigma_y$ & $D_m$ are uncorrelated. For this data: $b = 1.43$

3. Coeff. $a = mean(\sigma_y)$:

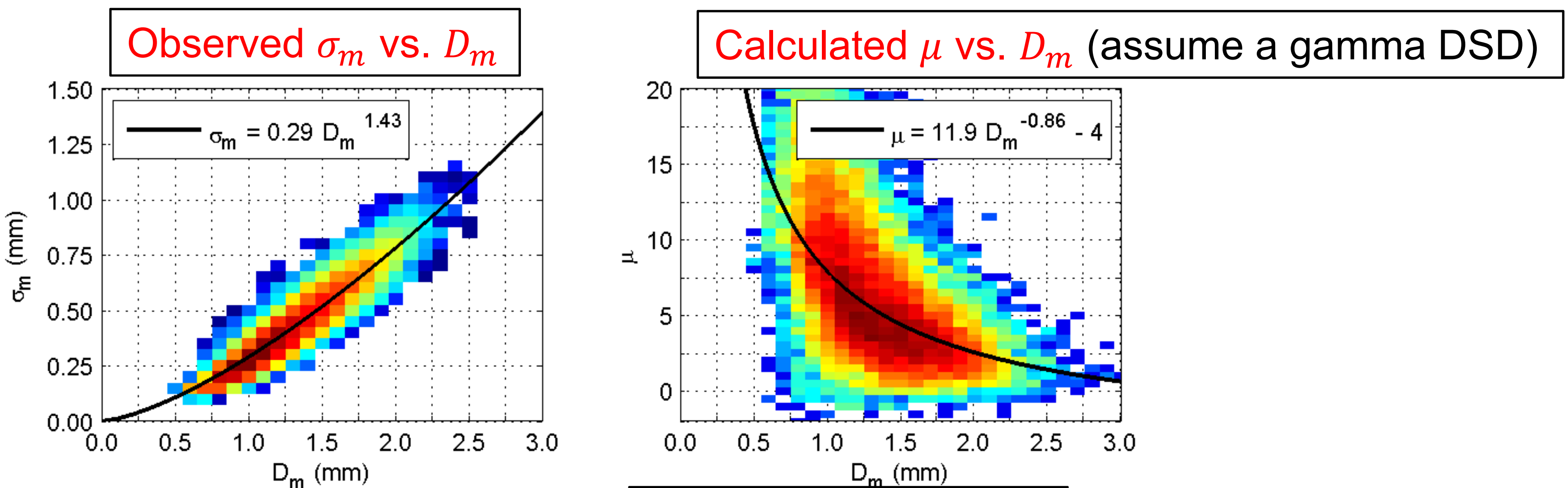
$$a = \overline{\sigma_y} = 0.29$$

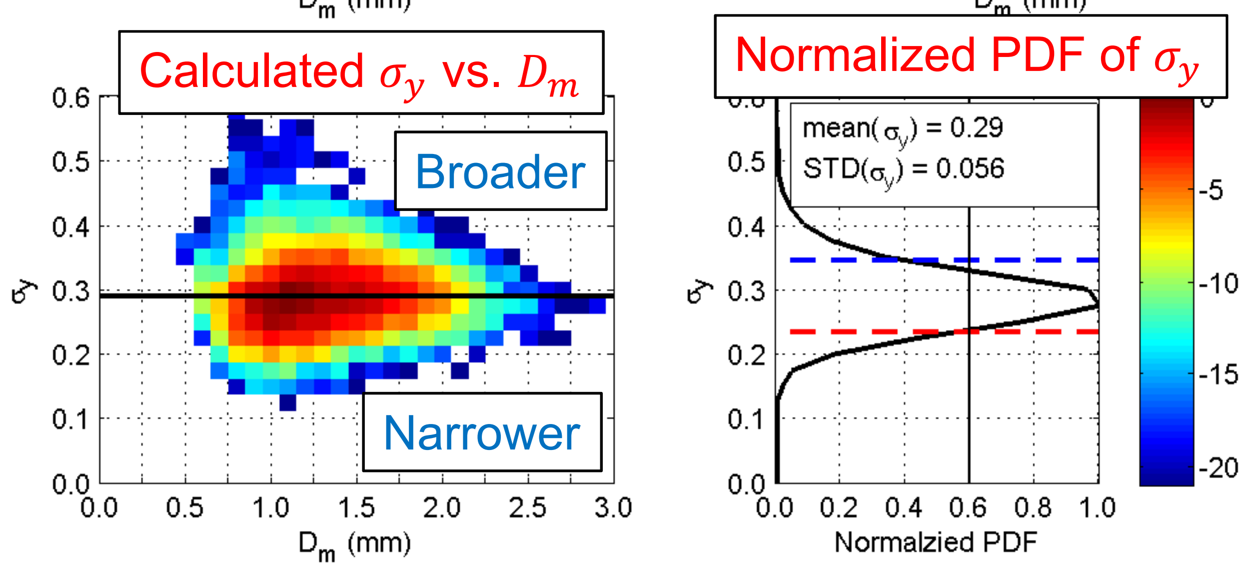
4. Power-law:

$$\sigma_m = \overline{\sigma_y} D_m^b = 0.29 D_m^{1.43}$$

Haddad et al. (1996)

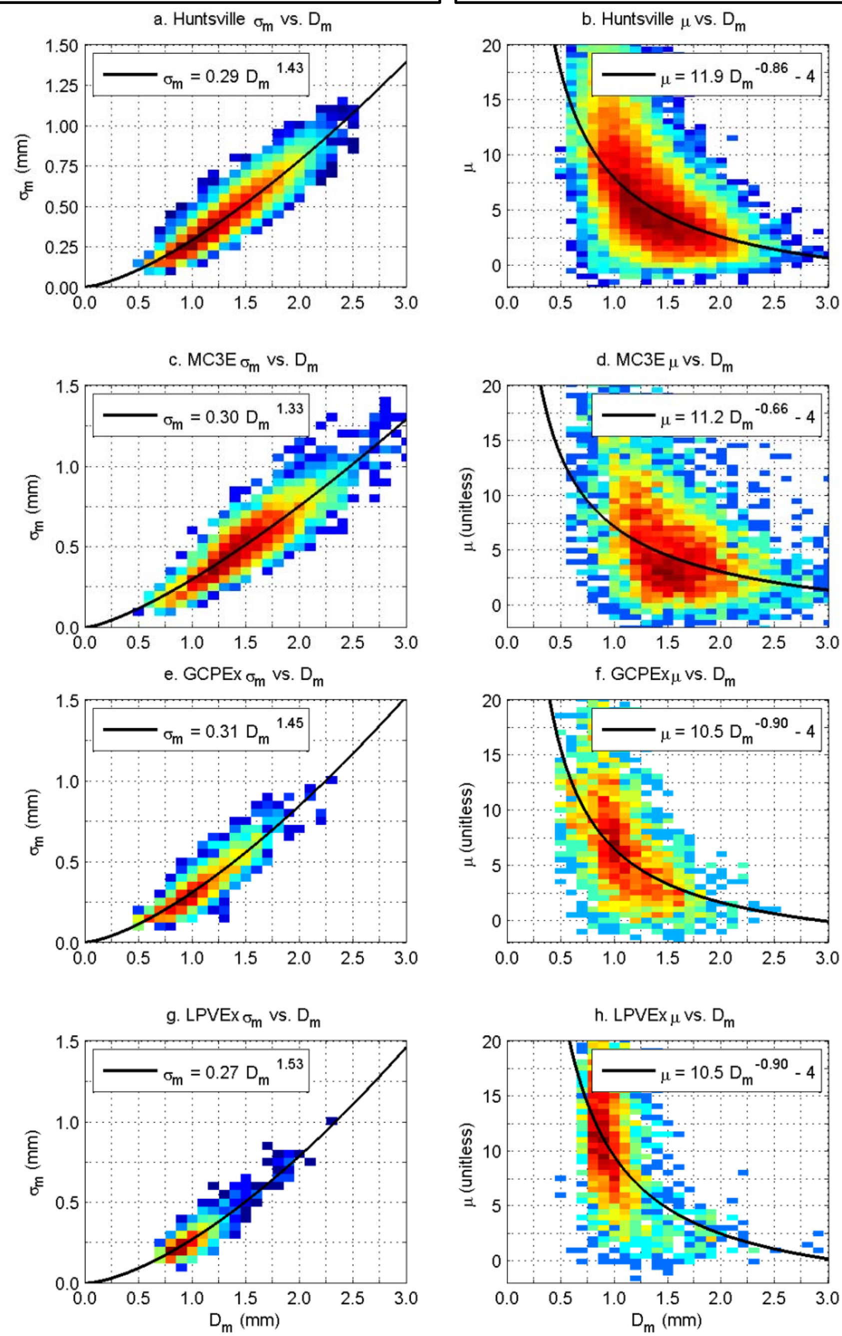
# Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples




74% of observations are within +/- 1 STD (a normal distribution would have 68%)

| Observed $\sigma_m$ vs. $D_m$ | Calculated $\mu$ vs. $D_m$ |
|---|---|




Huntsville:, 20,954 samples

$$\sigma_m = 0.29D_m^{1.43}$$

MC3E: 5,175 samples

$$\sigma_m = 0.30D_m^{1.33}$$

GCPEx: 2,218 samples

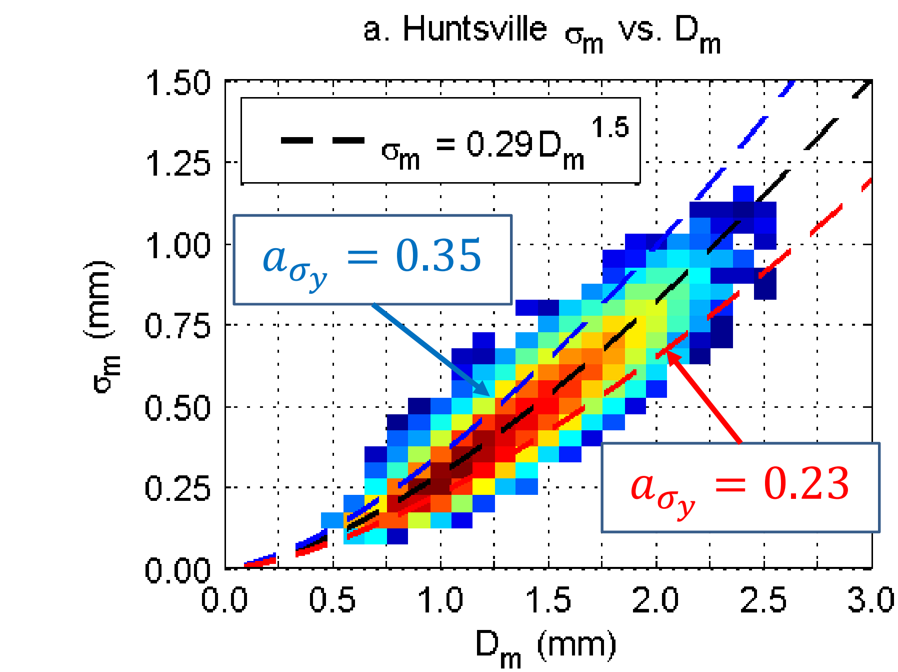
$$\sigma_m = 0.31D_m^{1.45}$$

LPVEx: 2,454 samples

$$\sigma_m = 0.27D_m^{1.53}$$

Ensemble: 29,555 samples

$$\sigma_m = 0.29D_m^{1.42}$$




# Adaptive Power-law Constraints for $\sigma_m - D_m$ and $\mu - D_m$

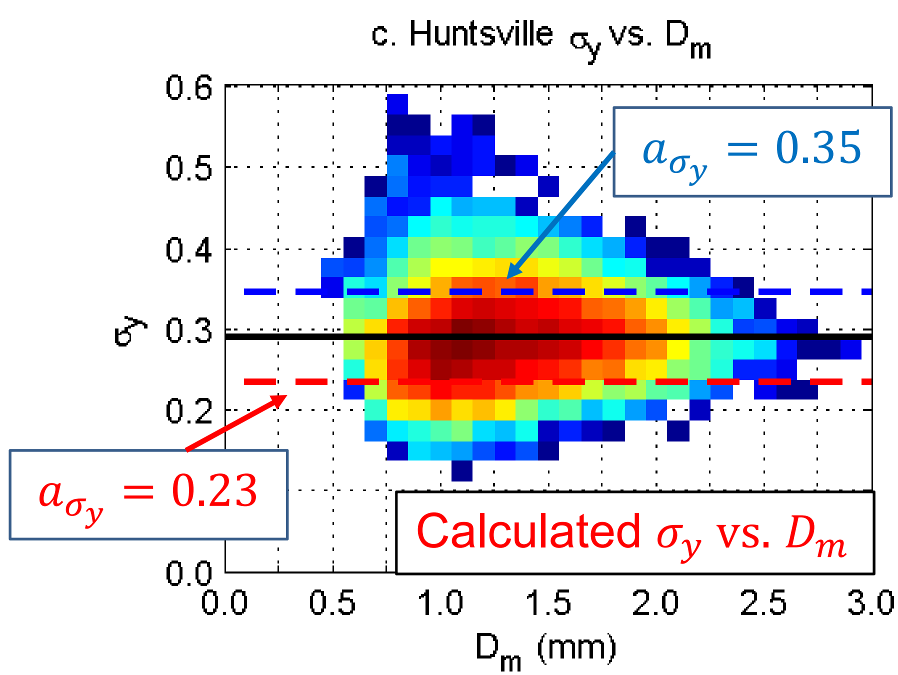
Observed $b$ ranged from 1.33 to 1.53.

By setting $b = 1.5$,

$$_{constraint}\sigma_m = a_{\sigma_y} D_m^{1.5}$$

- Constraint is only a function of $a_{\sigma_y}$
- $\mu - D_m$ constraint has a simple form:

$$_{constraint}\mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$$




Change $a_{\sigma_y}$ to get a different constraint.

$$_{constraint}\sigma_m = 0.35 D_m^{1.5} \Rightarrow \overline{\sigma_y} + std(\sigma_y)$$

$$_{constraint}\sigma_m = 0.29 D_m^{1.5} \Rightarrow \overline{\sigma_y} \text{ (best fit)}$$

$$_{constraint}\sigma_m = 0.23 D_m^{1.5} \Rightarrow \overline{\sigma_y} - std(\sigma_y)$$

# Discussion Points

The power-law relationship appears to be robust for rain observed at different locations.

The calculation of $D_m$ and $\sigma_m$Sm can be calculated for all raindrop distributions without assuming a shape of the distribution.

But this relationship raises many questions:

- How does rain regime determine the power-law coefficients?
- Or, does rain regime just move the observation around the 2-d $D_m - \sigma_m$ distribution?
- Do cloud droplet distributions have similar $D_m - \sigma_m$ power-law relationships?
  - Is there a temperature dependence?
- Are $D_m - \sigma_m$ power-law relationships a way to identify mixed phase clouds in ARM data?
- What are the 2-d distributions of $D_m$ and $\sigma_m$in cloud resolving models?
- Do 1-, 2-moment and bin microphysics modules capture $D_m - \sigma_m$ statistics?

These questions can be answered through collaboration between observational and model scientists.

# Concluding Remarks (1/2)

**Develop physically based relationships between DSD parameters**

- NASA GPM DSD Working Group is investigating relationships between DSD parameters to address assumptions used in retrieval algorithms.
- $\sigma_m \sim D_m^{1.5}$ relationship appears robust & observed in several field campaigns.
- Defined an adaptive constraint with one parameter: $\mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$
- Williams et al, 2013: Adaptive Raindrop Size Distribution Constraint for Probabilistic Rainfall Retrieval Algorithms, *submitted to J. Appl. Meteor. Climatol*.

**Develop a framework to incorporate GV findings into Algorithms**

- Divide Algorithm "Look-up Tables" into Scattering and Integral Tables.
- Scattering Tables describe the electromagnetic properties of particles
- Integral Tables describe particle size distributions

**Benefits** of dividing Look-up Tables into Scattering and Integral Tables:

1. Researchers can work independently – Developing scattering tables is independent of investigating particle size distributions.
2. Provides a framework to incorporate GV findings into Look-up Tables used by satellite algorithms.
3. Provides a communication framework for particle scattering modelers, observational scientists, and algorithm developers.