Drizzle measurements with the HSRL and the KAZR: sensitivity to assumptions

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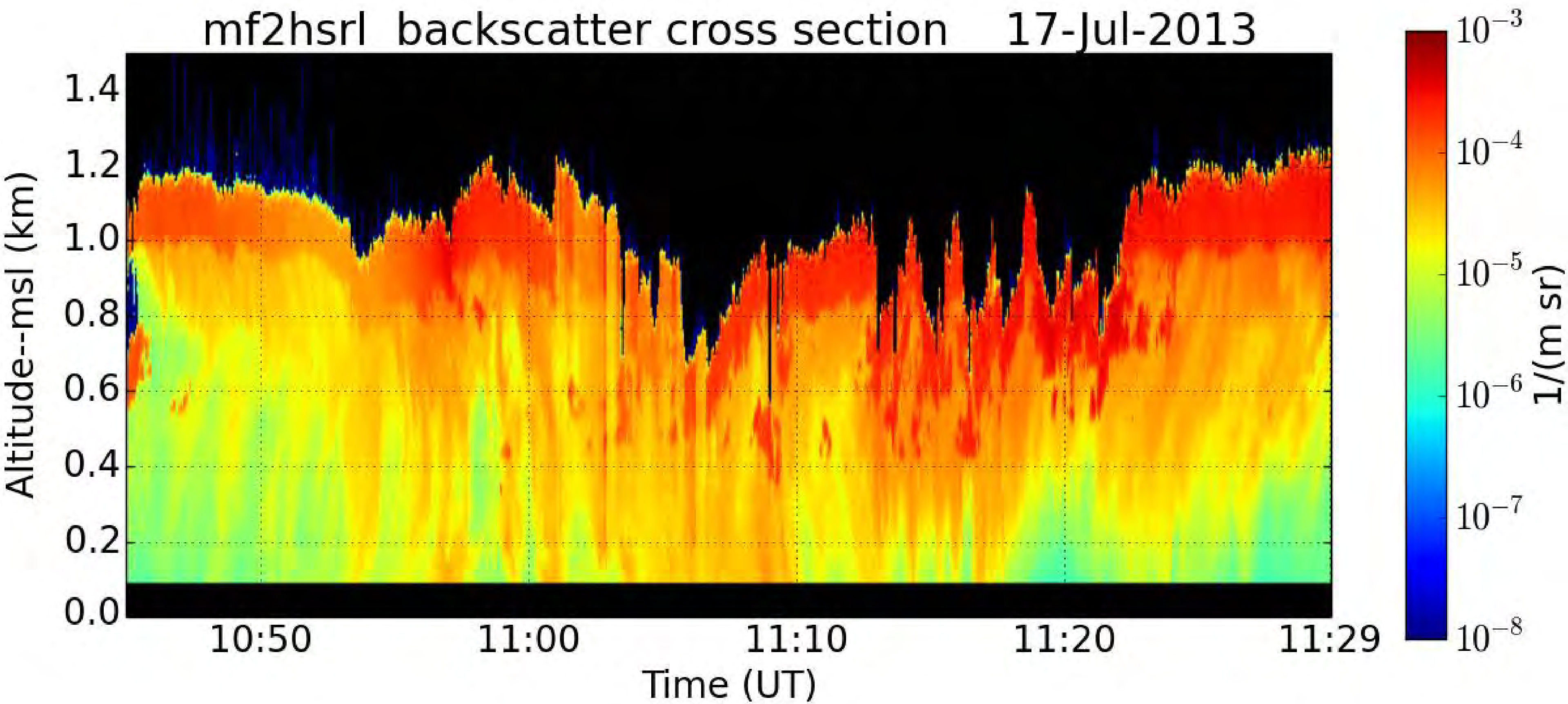
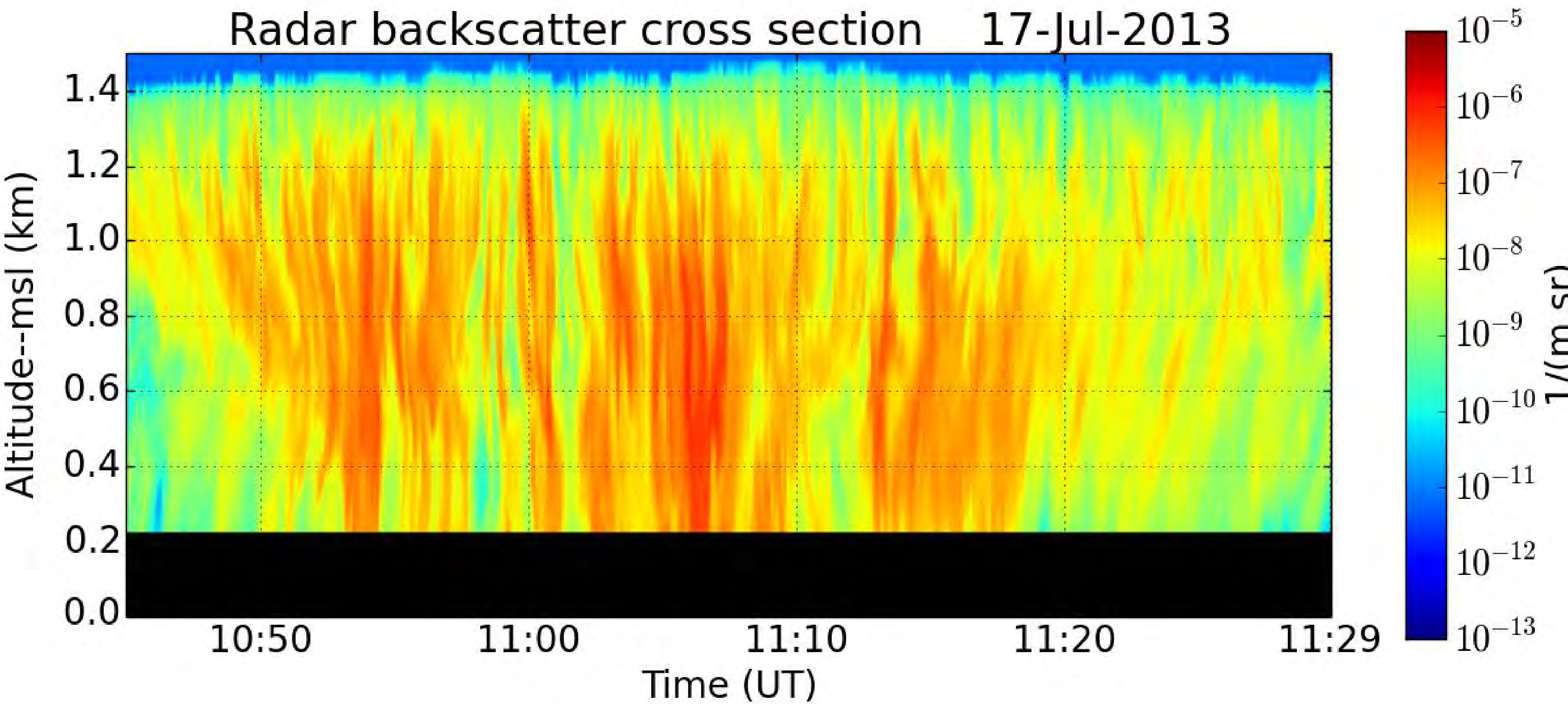
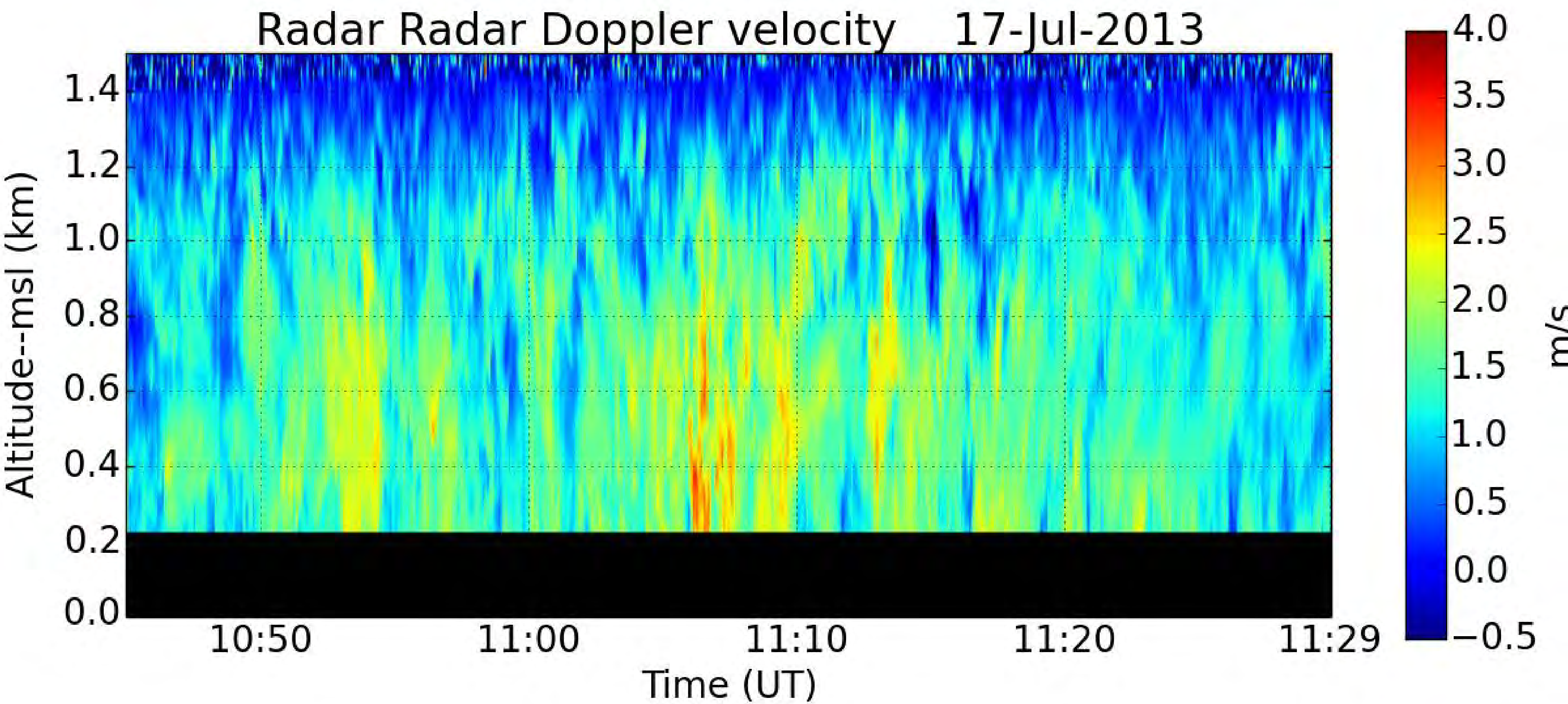


mf2hsrl backscatter cross section 17-Jul-2013
Altitude--msl (km)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10:50
11:00
11:10
11:20
11:29
Time (UT)
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
1/(m sr)


Radar backscatter cross section 17-Jul-2013
Altitude--msl (km)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
10:50
11:00
11:10
11:20
11:29
Time (UT)
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
1/(m sr)


Radar Radar Doppler velocity 17-Jul-2013
Altitude--msl (km)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
10:50
11:00
11:10
11:20
11:29
Time (UT)
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
m/s

HSRL-Radar particle size measurement

For droplets which are small compared to the radar wavelength but large compared to the lidar wavelength:

Radar scattering cross section = $\beta_{radar} \sim \langle V^2 \rangle \sim \langle D^6 \rangle$ ------- Rayleigh scattering
Lidar extinction cross section = $\beta_{lidar} \sim \langle A \rangle \sim \langle D^2 \rangle$ ------- Geometric optics

Where:

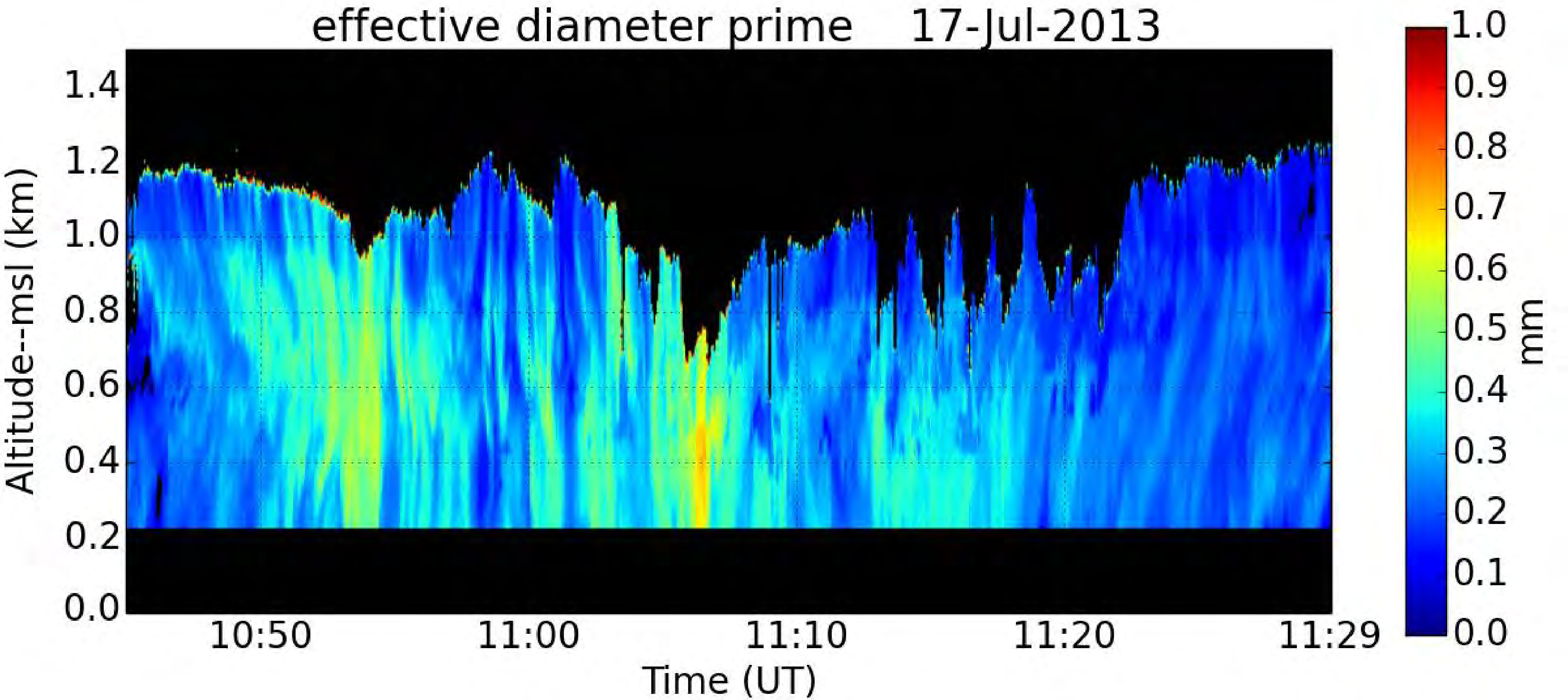
$\langle V^2 \rangle$ = average volume squared of the particles
$\langle A \rangle$ = average projected area of the particles

We can define:

$$D'_{eff} = \left(\frac{\lambda_{radar}}{\pi}\right) \cdot \left(\frac{3 \cdot \beta_{radar}}{4 \cdot k_w^2 \cdot \beta_{lidar}}\right)^{\frac{1}{4}}$$

Where:

λ_{radar} = radar wavelength
k_w = dielectric constant of water


effective diameter prime 17-Jul-2013
Altitude--msl (km)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10:50
11:00
11:10
11:20
11:29
Time (UT)
mm
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0

Assuming a gamma distribution of particle diameters:

$$N = a\int_0^\infty D^\alpha \cdot exp(-\frac{\alpha}{\gamma} \cdot (\frac{D}{D_m})^\gamma) \cdot dD$$

Where:

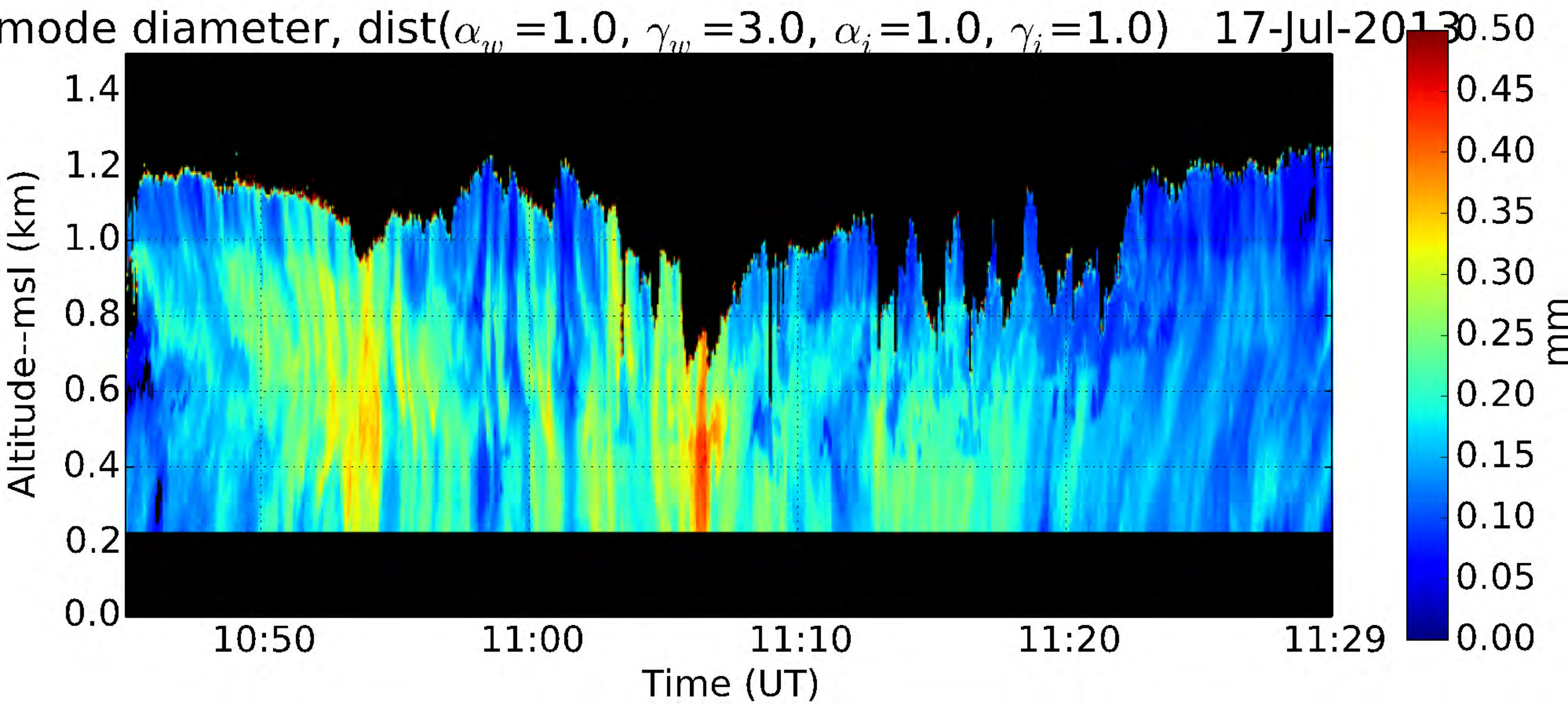
- N = number of particles
- D = particlle Diameter
- D_m= mode diameter

The effective diameter prime can be written as:

$$D'_{eff} = \frac{\int D^{\alpha+6} \cdot exp(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}{}^\gamma) \cdot dD}{\int D^{\alpha+2} \cdot exp(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}{}^\gamma) \cdot dD} = \frac{\frac{\alpha}{\gamma}^{\frac{1}{\gamma}} \cdot D_m \cdot \Gamma(\frac{\alpha+7}{\gamma})}{\Gamma(\frac{\alpha+3}{\gamma})}$$

Solving for the mode diameter, D_m:

$$D_m = D'_{eff} \cdot (\frac{\alpha}{\gamma})^{\frac{1}{\gamma}} \cdot (\frac{\Gamma(\frac{\alpha+3}{\gamma})}{\Gamma(\frac{\alpha+7}{\gamma})})^{\frac{1}{4}}$$



mode diameter, dist(α_w =1.0, γ_w =3.0, α_i =1.0, γ_i =1.0) 17-Jul-2013
Altitude--msl (km)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10:50
11:00
11:10
11:20
11:29
Time (UT)
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
mm

The radar weighted fall velocity, $<V_{rf}>$:

$$< V_{rf} >= \frac{\int V_f \cdot D^{\alpha+6} \cdot exp(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^{\gamma}) \cdot dD}{\int D^{\alpha+6} \cdot exp(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^{\gamma}) \cdot dD}$$

And the mass weighted fall velocity, $<V_{mf}>$:

$$< V_{mf} >= \frac{\int V_f \cdot D^{\alpha+3} \cdot exp(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^{\gamma}) \cdot dD}{\int D^{\alpha+3} \cdot exp(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^{\gamma}) \cdot dD}$$

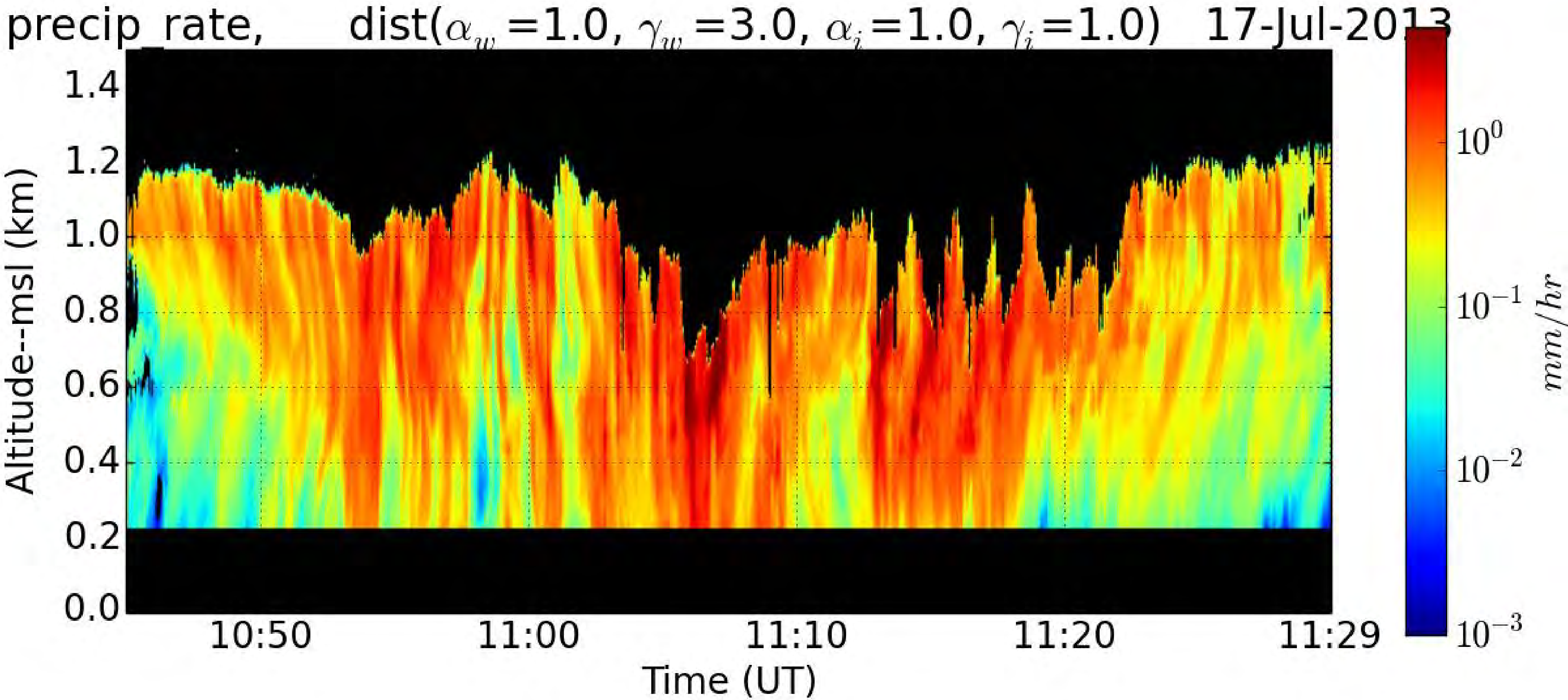
Where the fall velocity, V_f , is computed from:

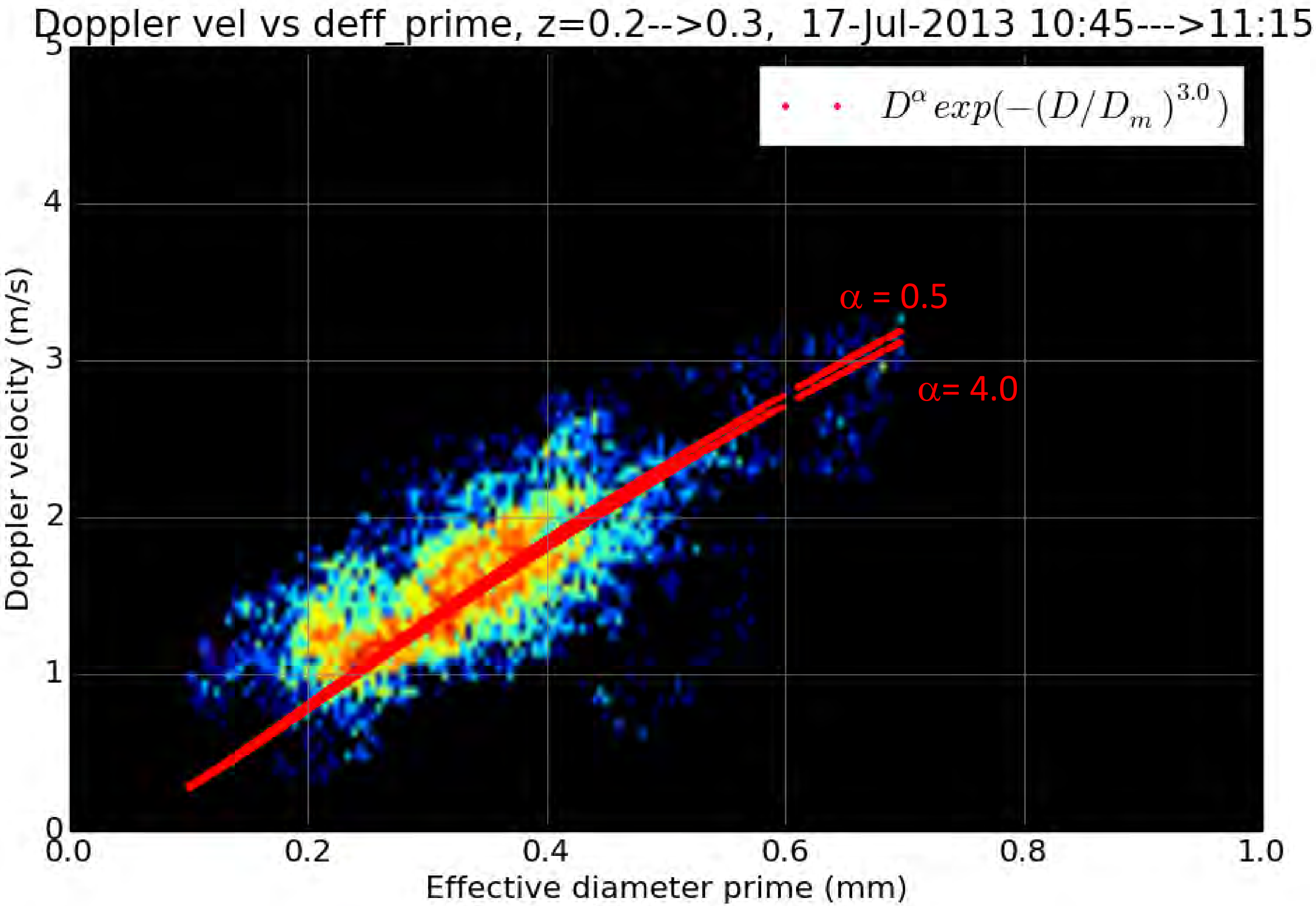
$$V_f = \frac{\eta}{\rho_{air} D} \left(\frac{\delta_0^2}{4} [(1 + C_1 X^{\frac{1}{2}})^{\frac{1}{2}} - 1]^2 - a_0 X^{b_0} \right)$$

Khovostyanov and Curry, JAS May 2005, Vol 62

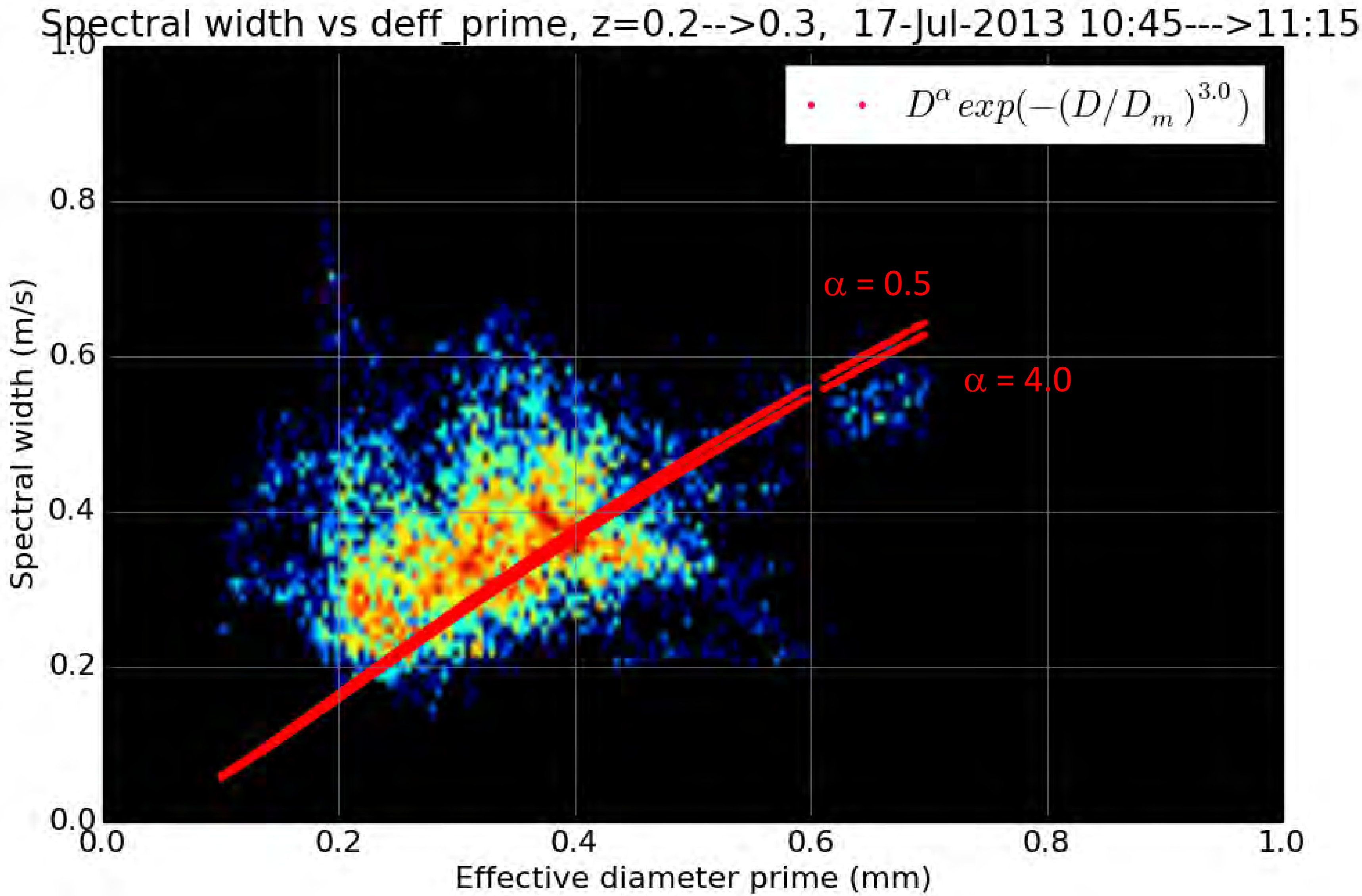
And the Beard Number, X expressed in terms of particle area, volume and density along with the acceleration of gravity, air density, particle diameter and the dynamic viscosity:

$$X = \frac{2 \cdot volume \cdot \rho_{particle} \cdot \rho_{air} \cdot g \cdot D^2}{area \cdot \eta^2}$$

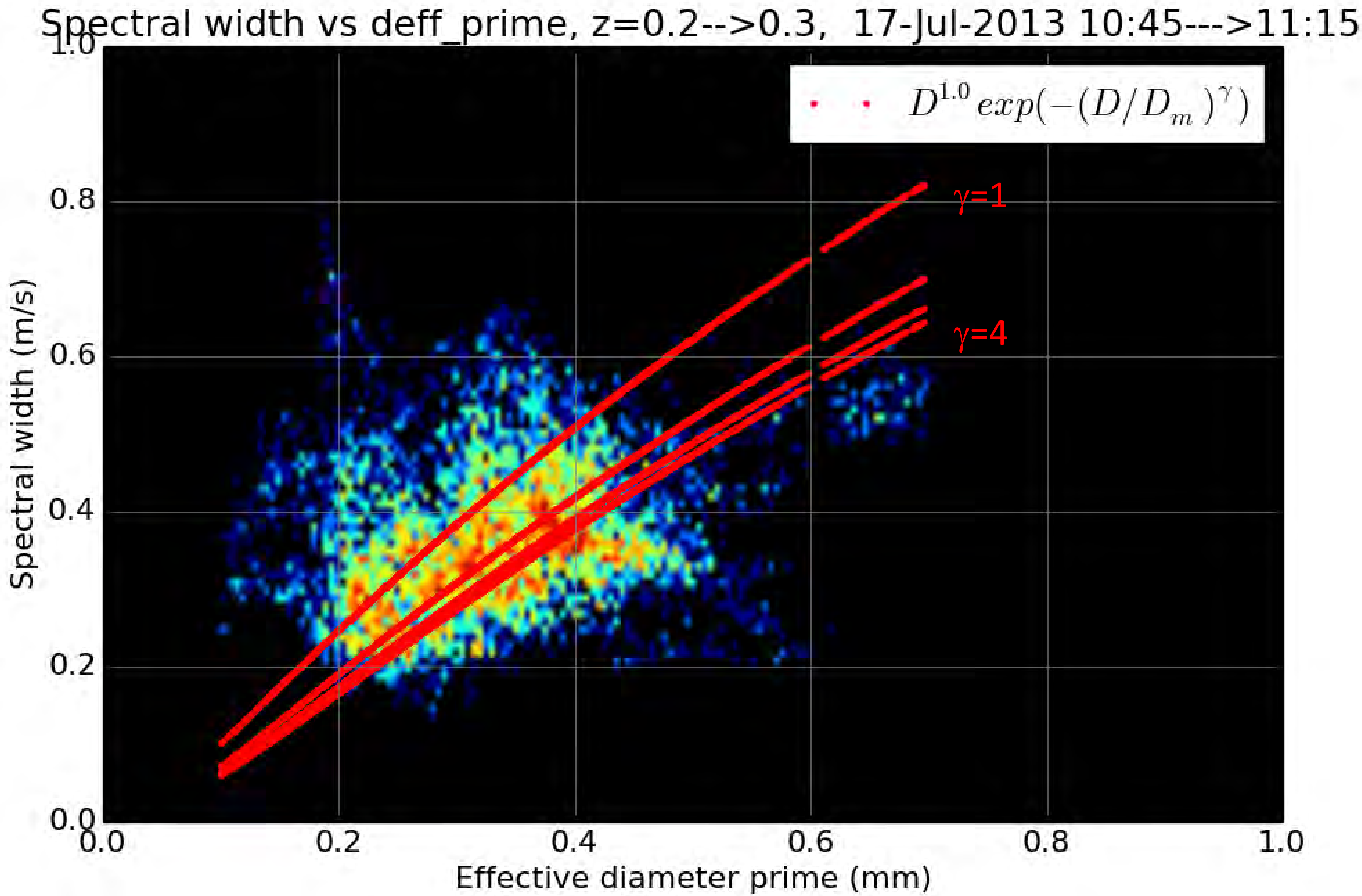

precip_rate, dist(α_w=1.0, γ_w=3.0, α_i=1.0, γ_i=1.0) 17-Jul-2013
Altitude--msl (km)
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
10:50
11:00
11:10
11:20
11:29
Time (UT)
mm/hr
10^0
10^{-1}
10^{-2}
10^{-3}



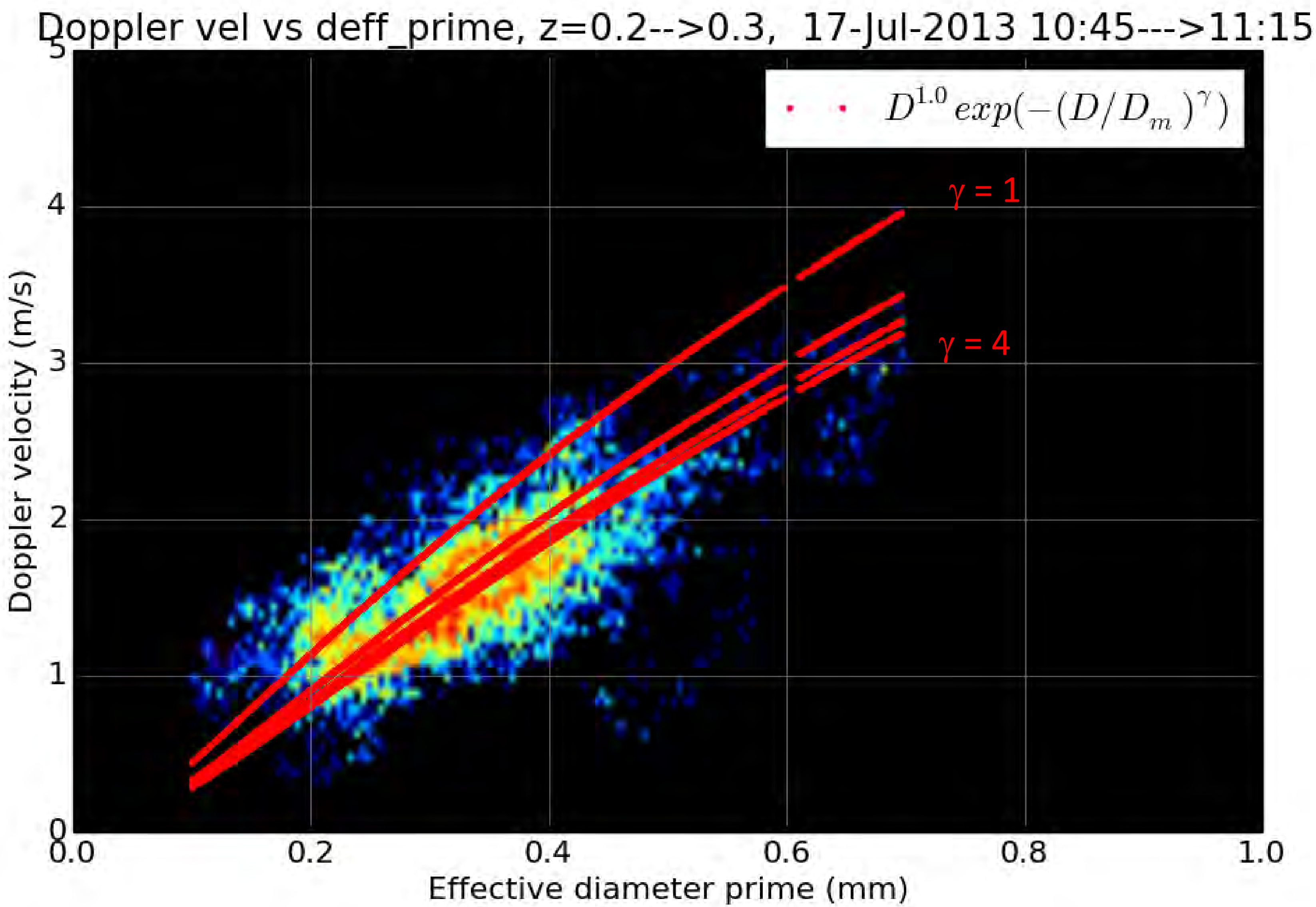
Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha}\, exp(-(D/D_m)^{3.0})$
α = 0.5
α= 4.0
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0



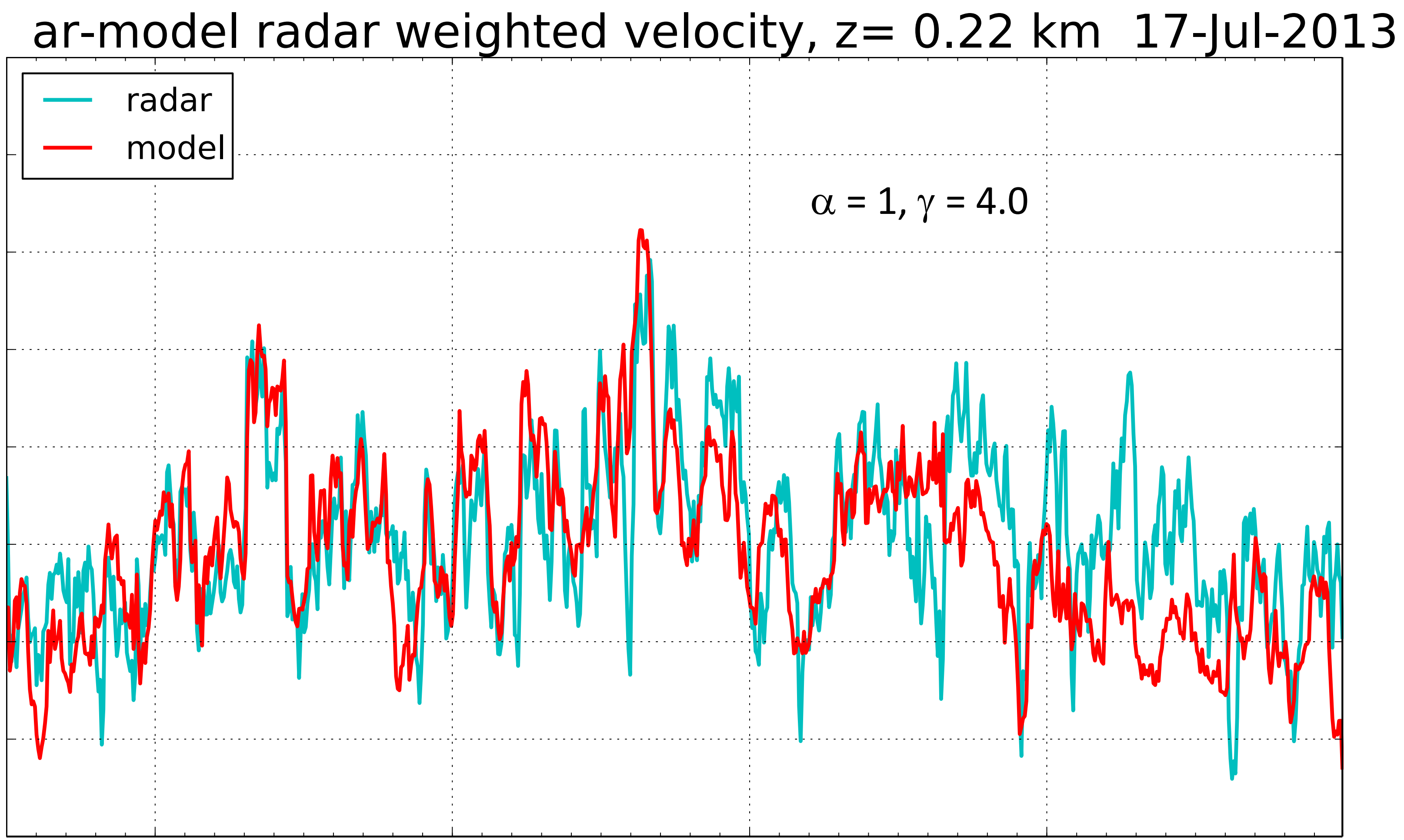
Spectral width vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha}\, exp(-(D/D_m)^{3.0})$
α = 0.5
α = 4.0
Spectral width (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0

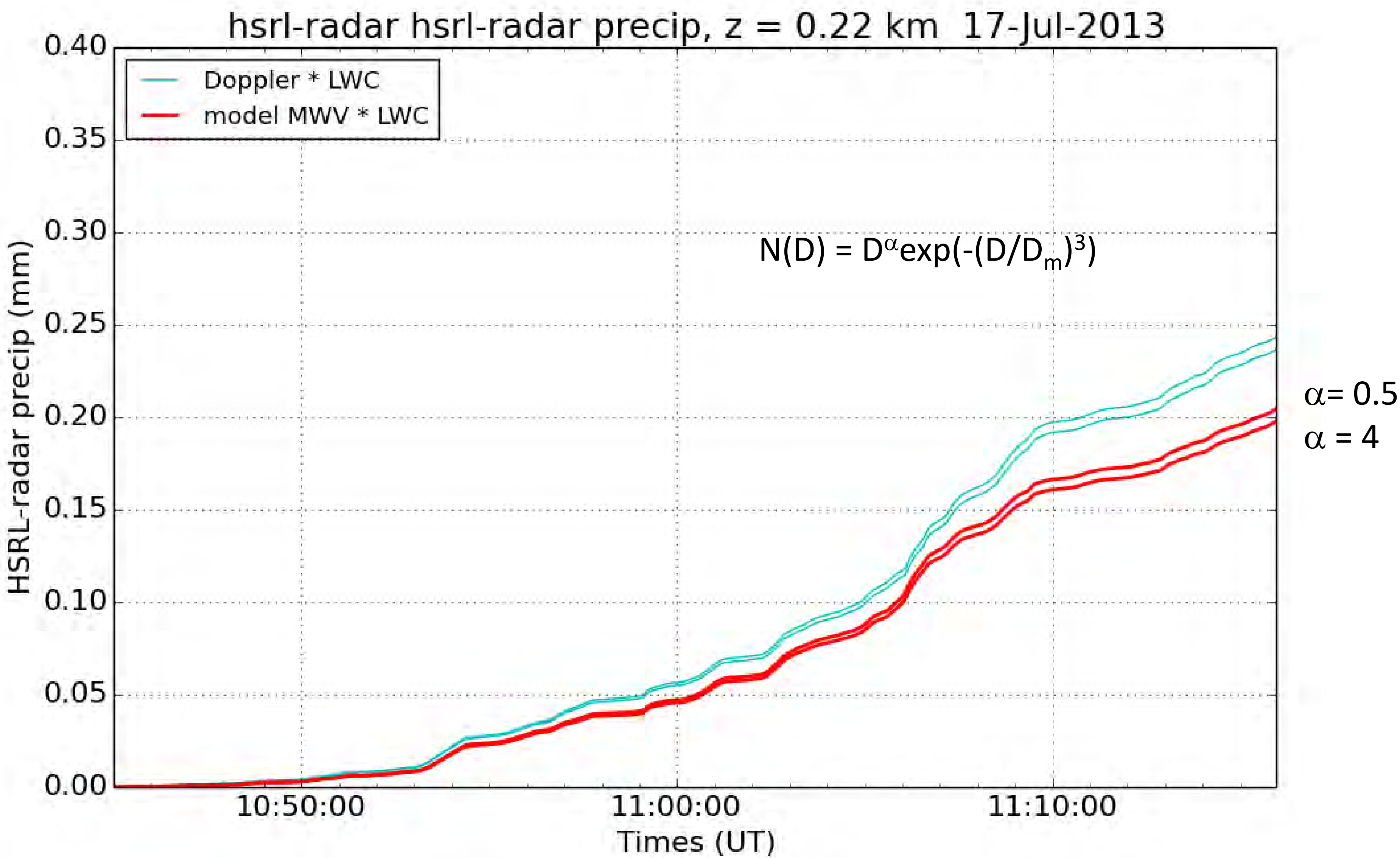

Spectral width vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{1.0} exp(-(D/D_m)^{\gamma})$
γ=1
γ=4
Spectral width (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0

Gamma = 1, 2, 3, 4

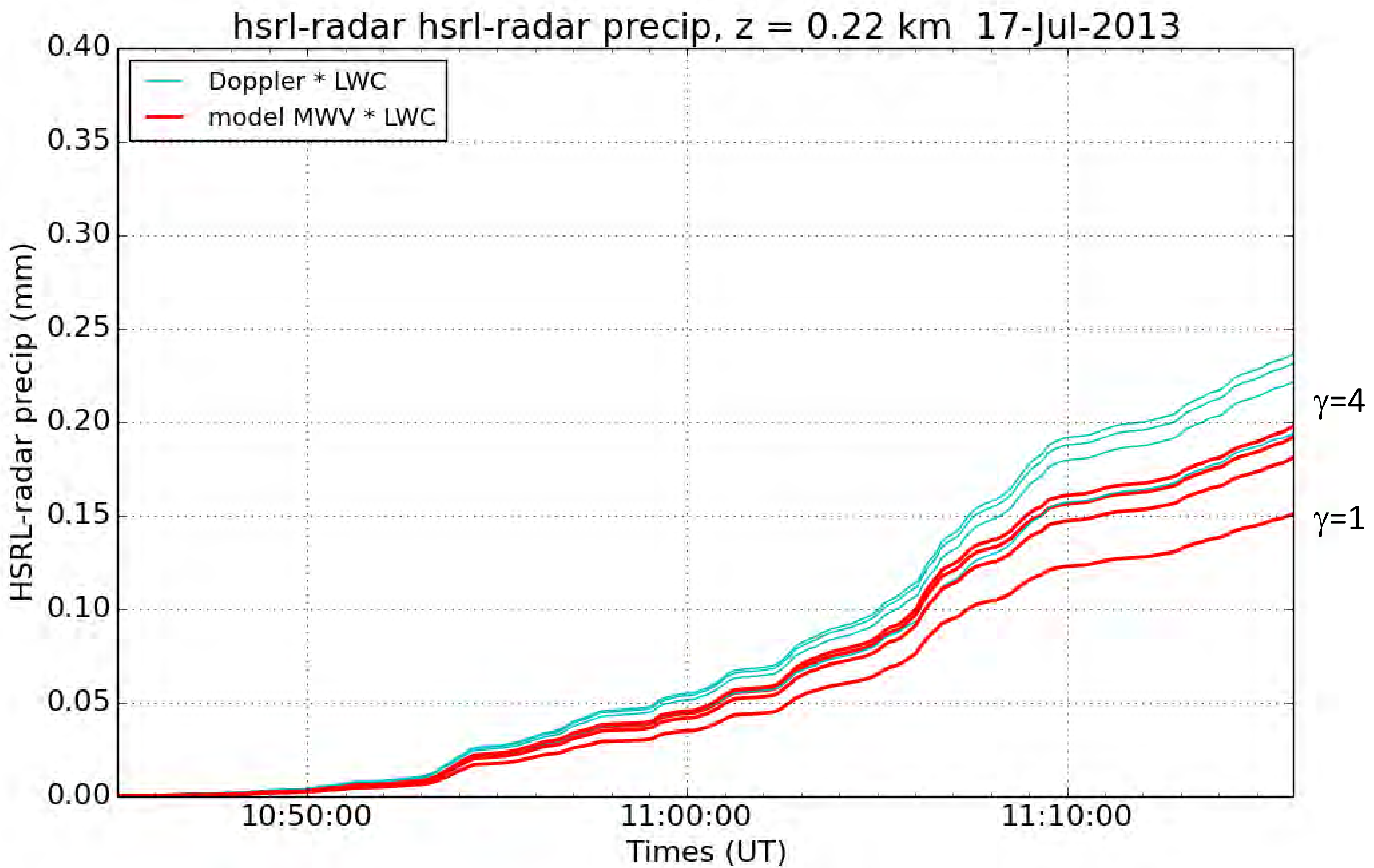


Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{1.0} exp(-(D/D_m)^\gamma)$
γ = 1
γ = 4
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0

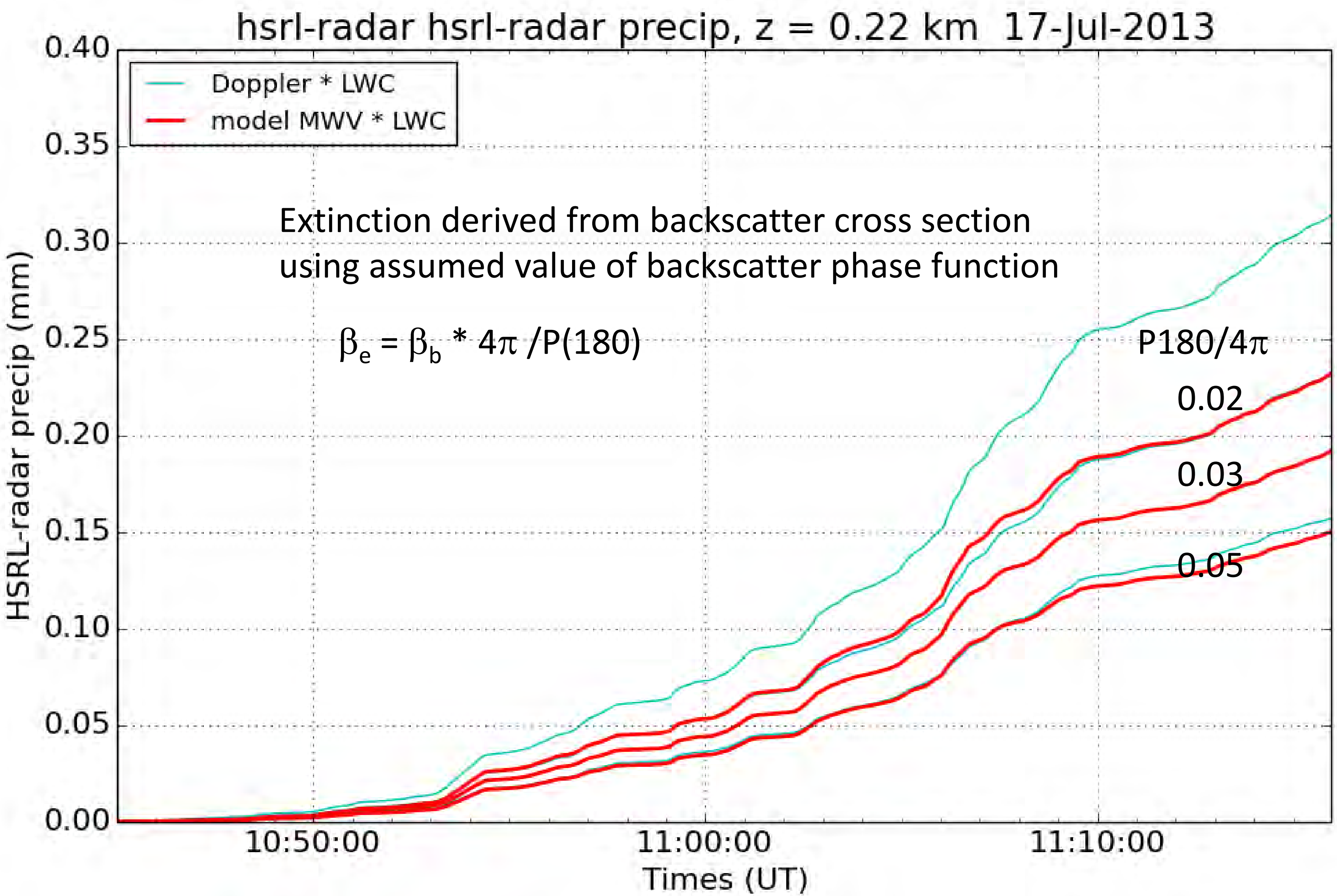

ar-model radar weighted velocity, z= 0.22 km 17-Jul-2013
radar
model
α = 1, γ = 4.0



hsrl-radar hsrl-radar precip, z = 0.22 km 17-Jul-2013
Doppler * LWC
model MWV * LWC
N(D) = D^α exp(-(D/D_m)^3)
α= 0.5
α = 4
HSRL-radar precip (mm)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
10:50:00
11:00:00
11:10:00
Times (UT)

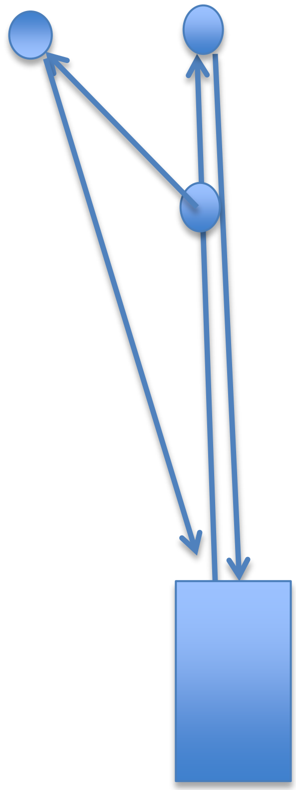


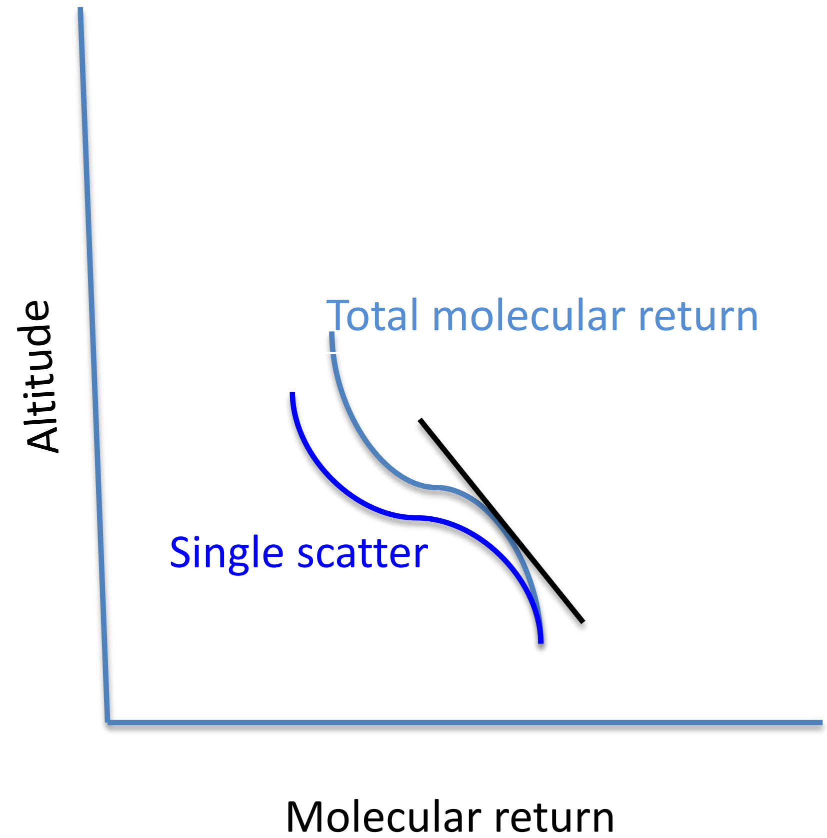
hsrl-radar hsrl-radar precip, z = 0.22 km 17-Jul-2013
Doppler * LWC
model MWV * LWC
HSRL-radar precip (mm)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
10:50:00
11:00:00
11:10:00
Times (UT)
γ=4
γ=1



hsrl-radar hsrl-radar precip, z = 0.22 km 17-Jul-2013
Doppler * LWC
model MWV * LWC
Extinction derived from backscatter cross section
using assumed value of backscatter phase function
$\beta_e = \beta_b * 4\pi / P(180)$
P180/4π
0.02
0.03
0.05
HSRL-radar precip (mm)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
10:50:00
11:00:00
11:10:00
Times (UT)

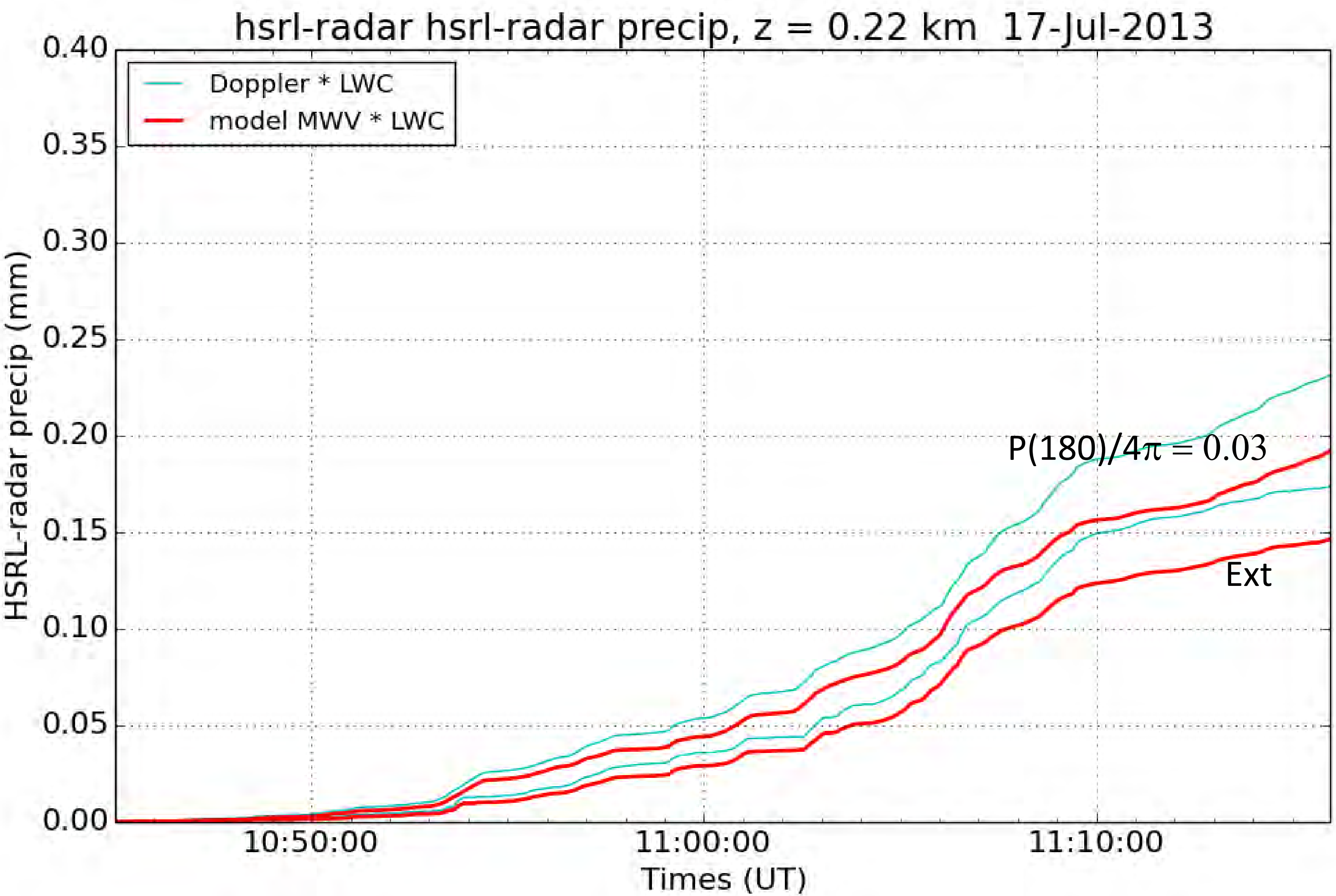
Extinction is derived from the slope of the molecular return
Multiple scattering reduces apparent extinction





Diffraction peak $\theta = \lambda/D \sim 0.5$ micron/500 micron = 1 mr

Extinction derived from p(180)/4p vs direct extinction measurment



Lidar ratio = 4π/P(180)

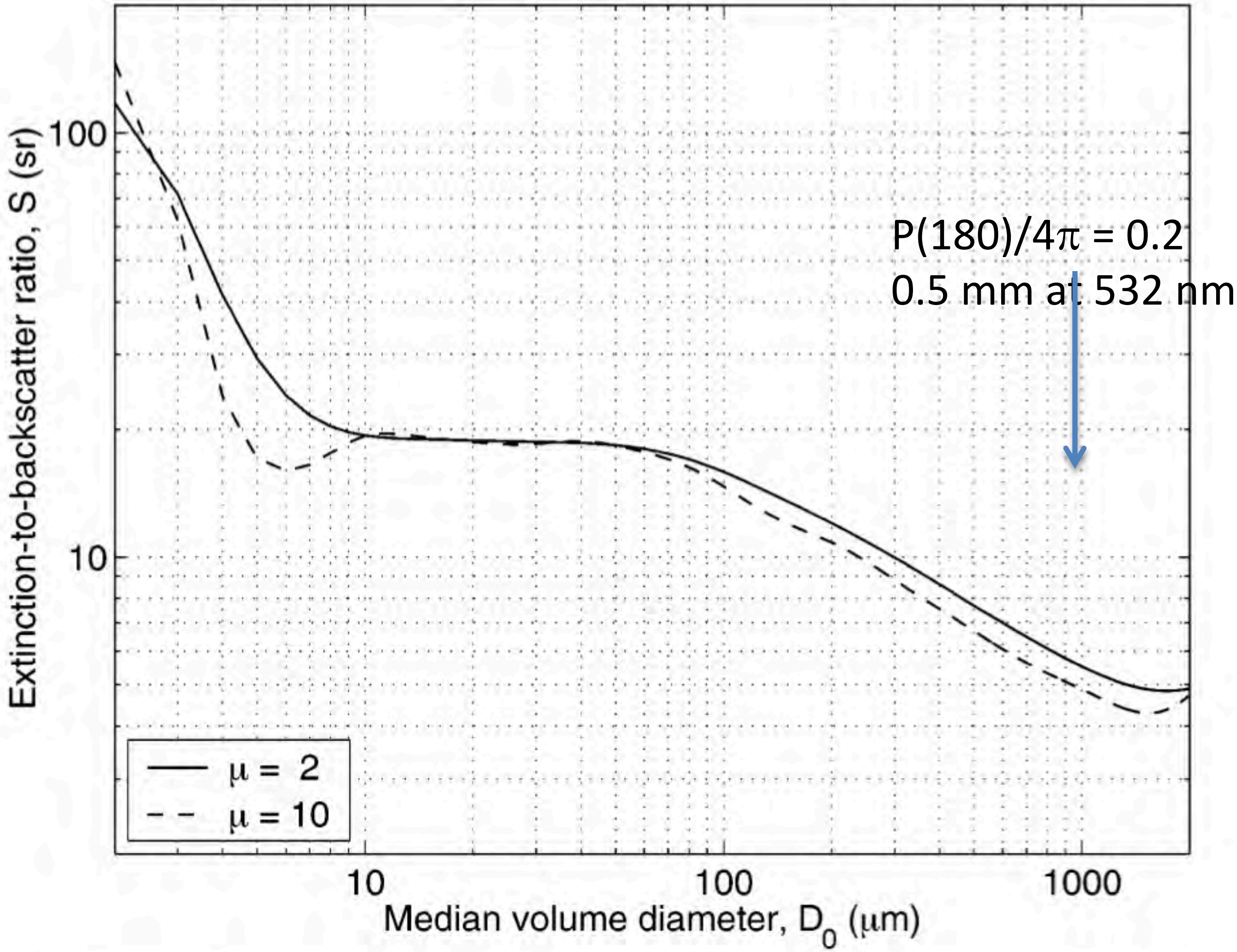
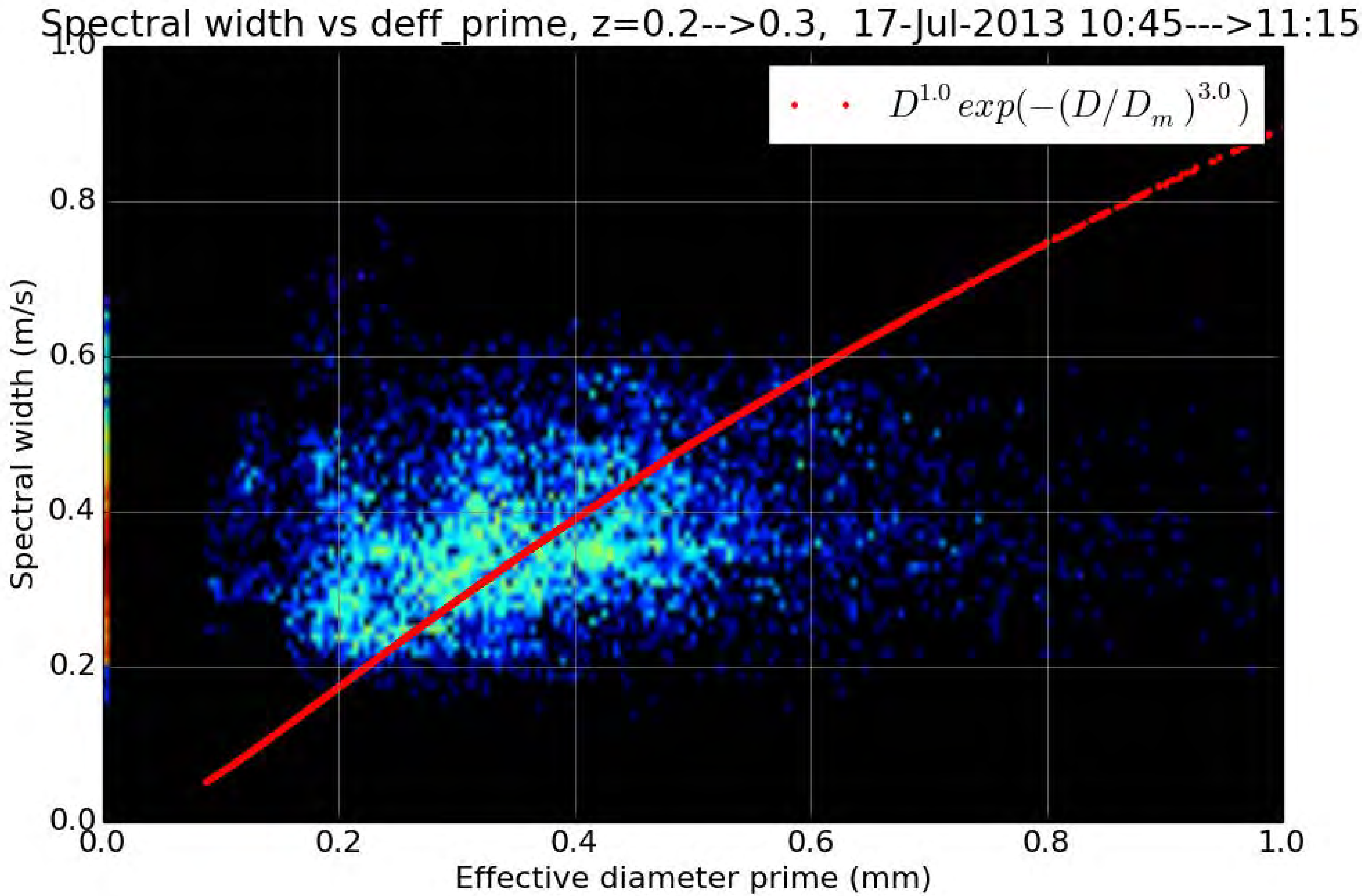
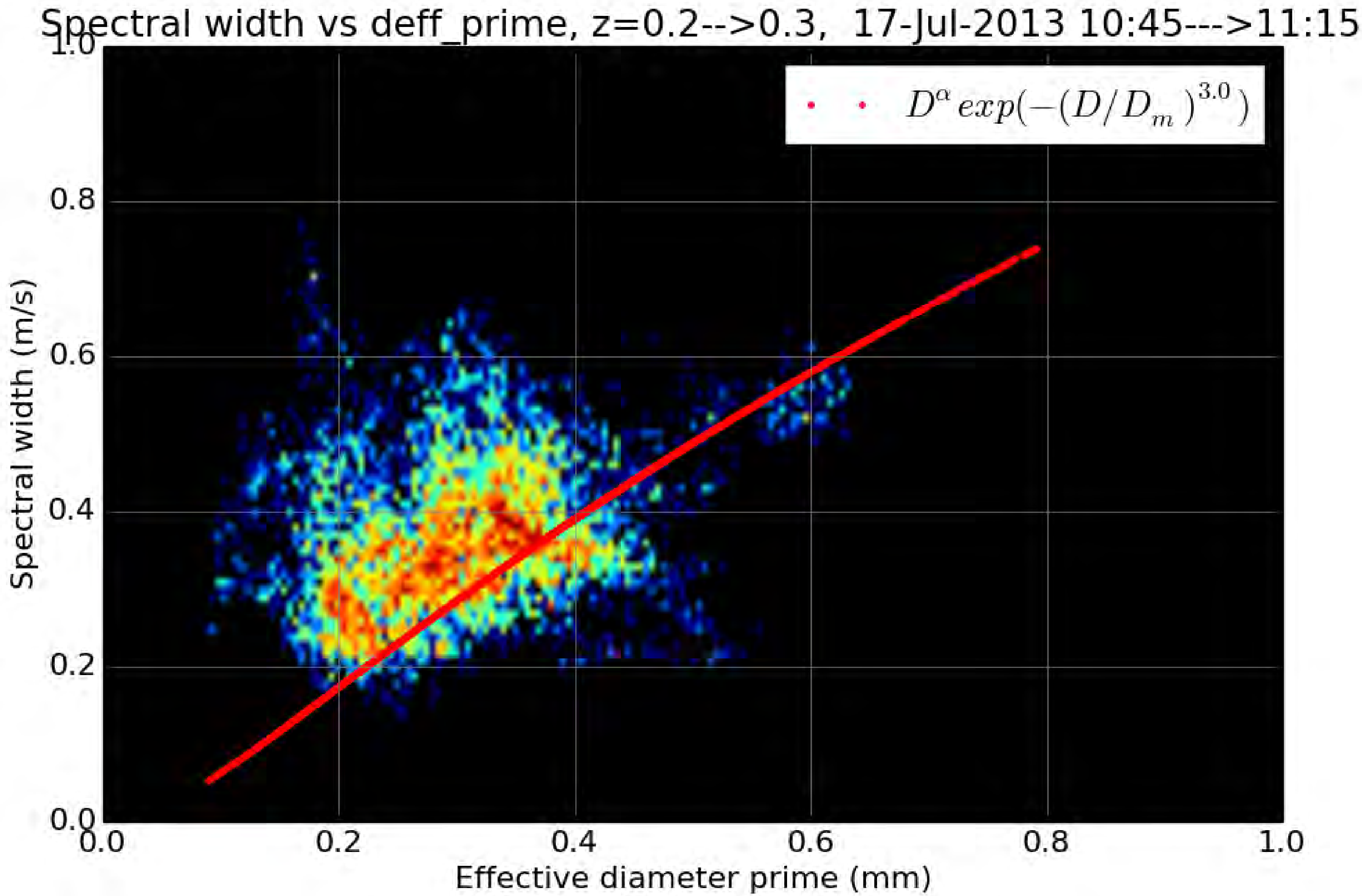


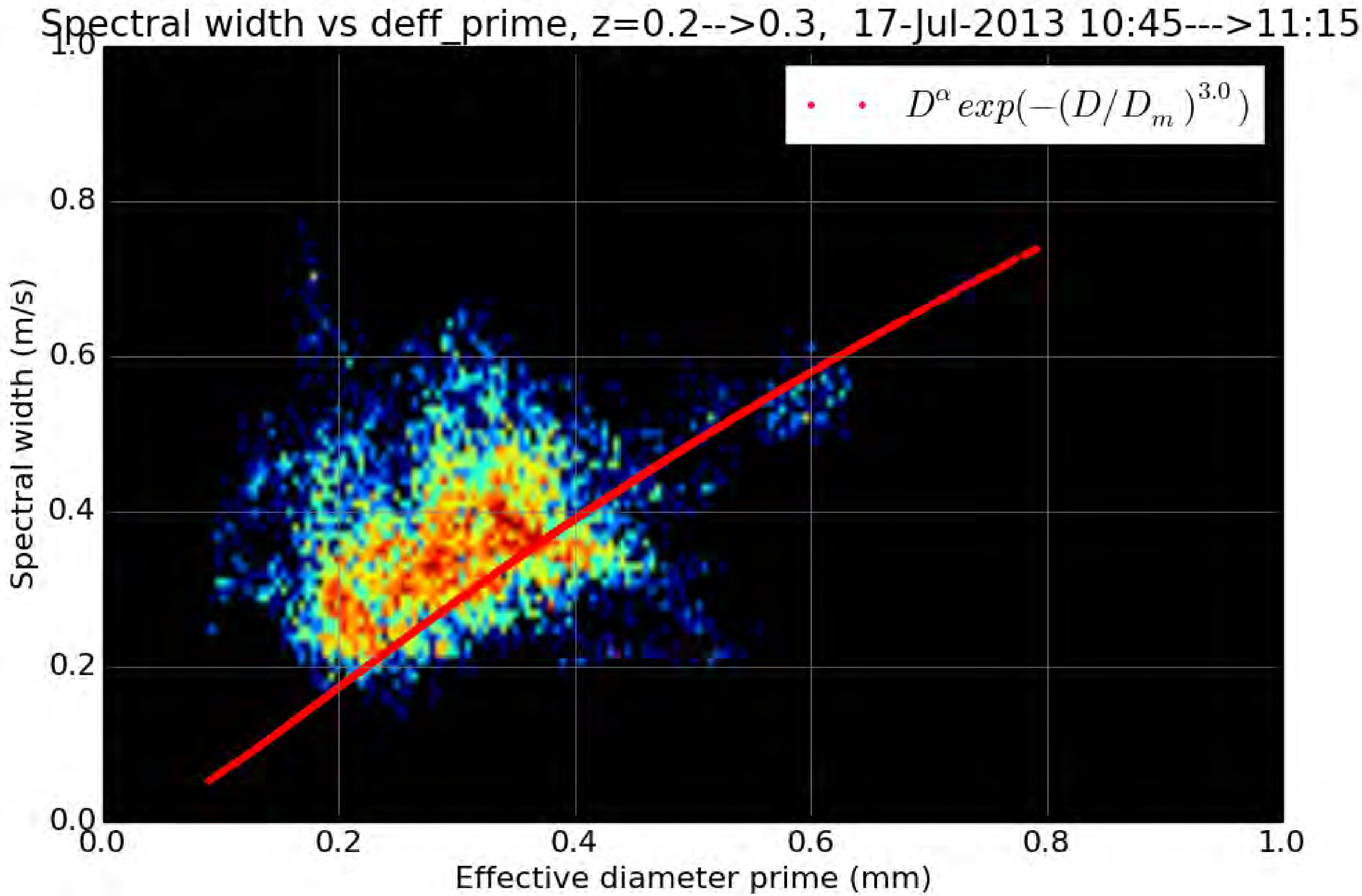
FIG. 3. Theoretical lidar ratio, S, at 905 nm as a function of median volume diameter for gamma distributions of droplet sizes with two different values of μ.



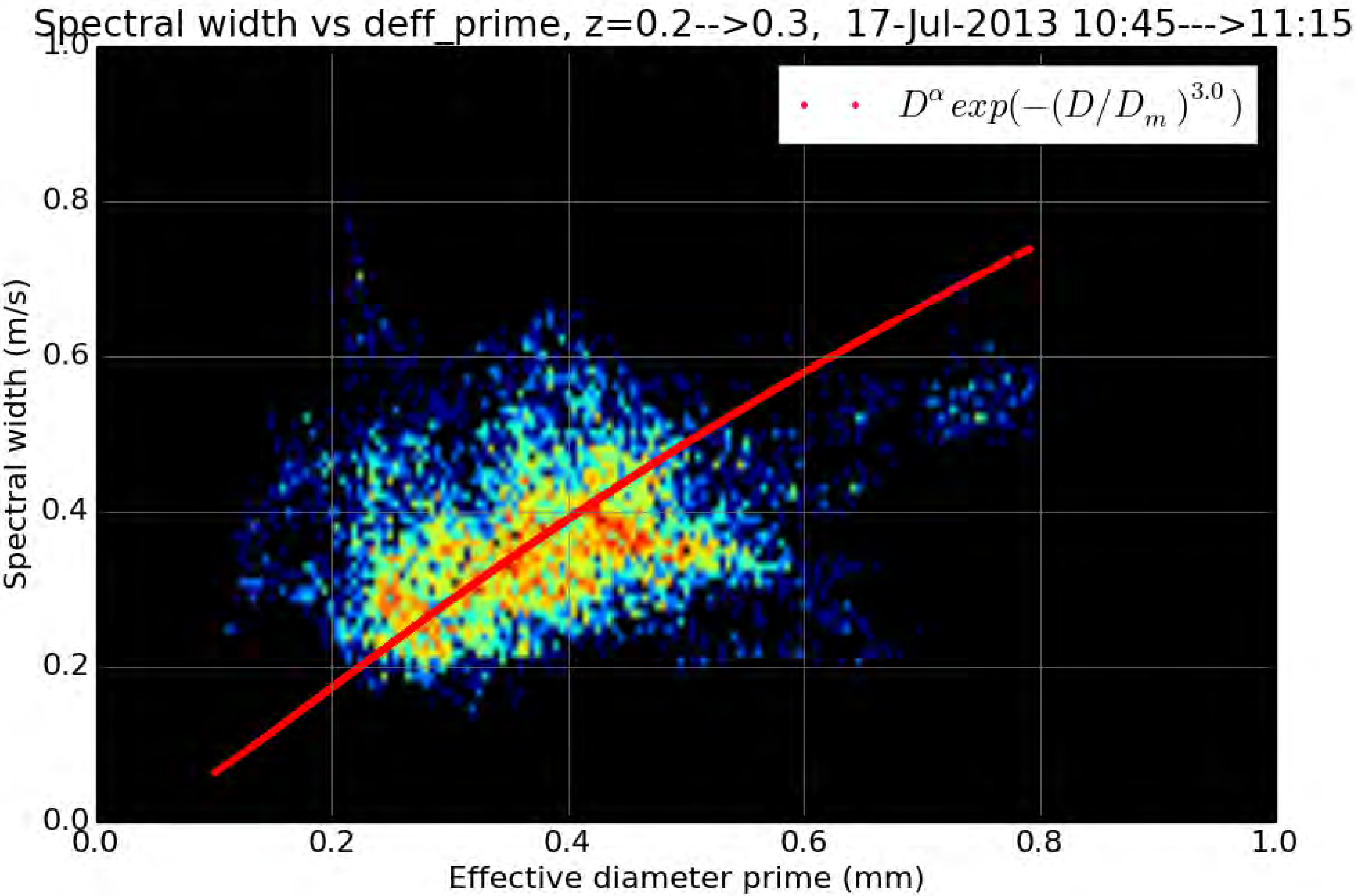
Spectral width vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{1.0} exp(-(D/D_m)^{3.0})$
Spectral width (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0



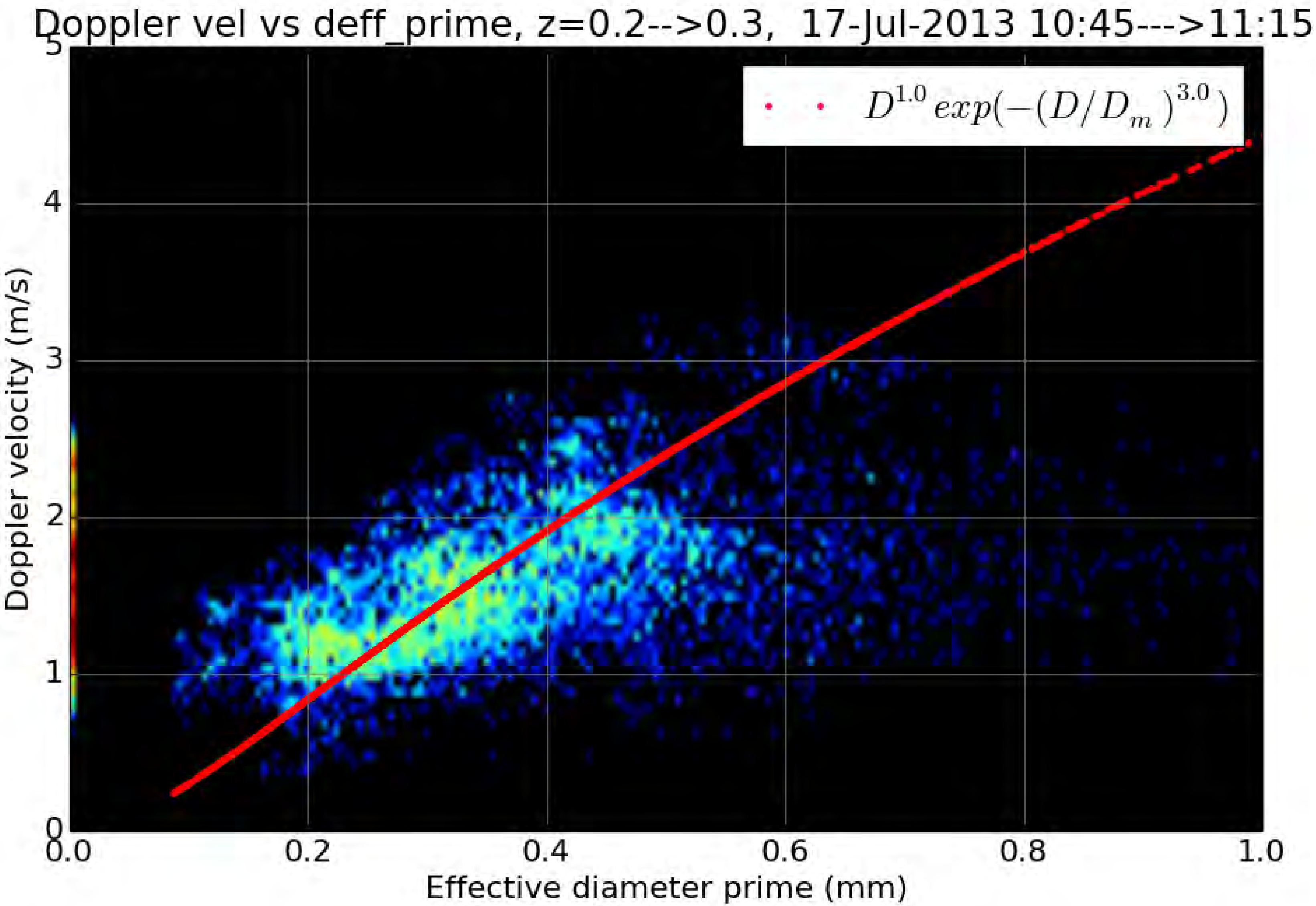
Spectral width vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha} exp(-(D/D_m)^{3.0})$
Spectral width (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0

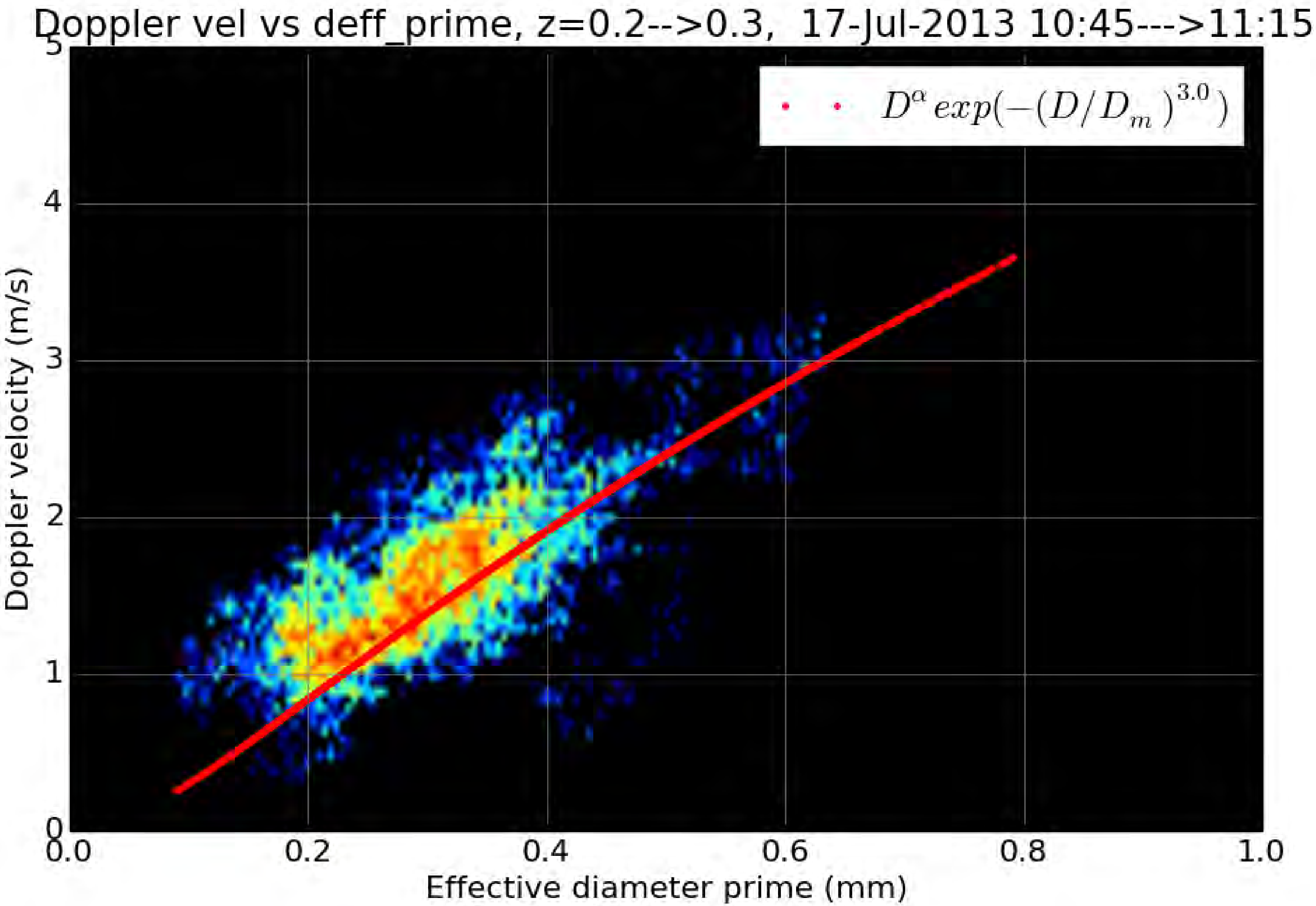


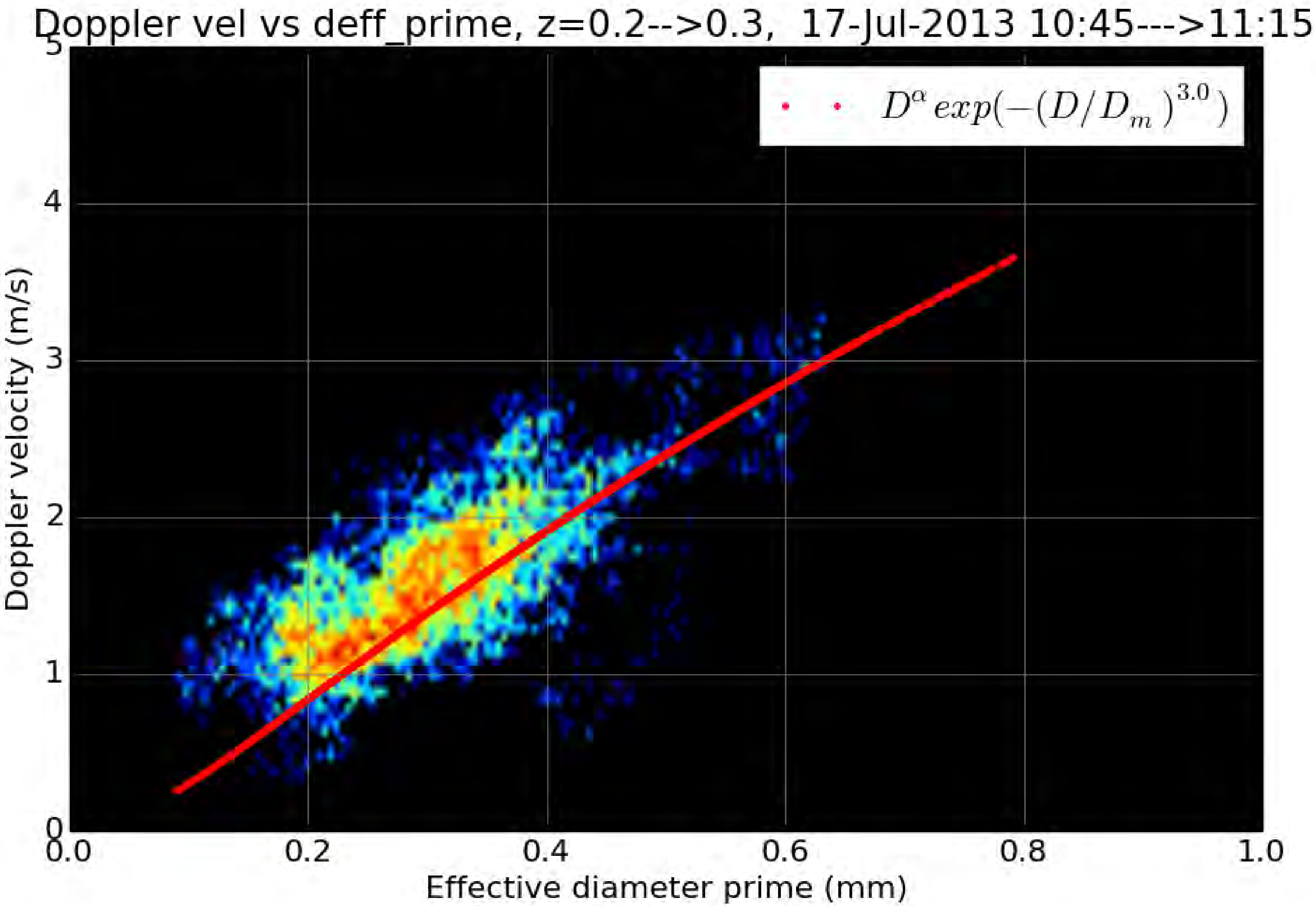
Spectral width vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha} exp(-(D/D_m)^{3.0})$
Spectral width (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0

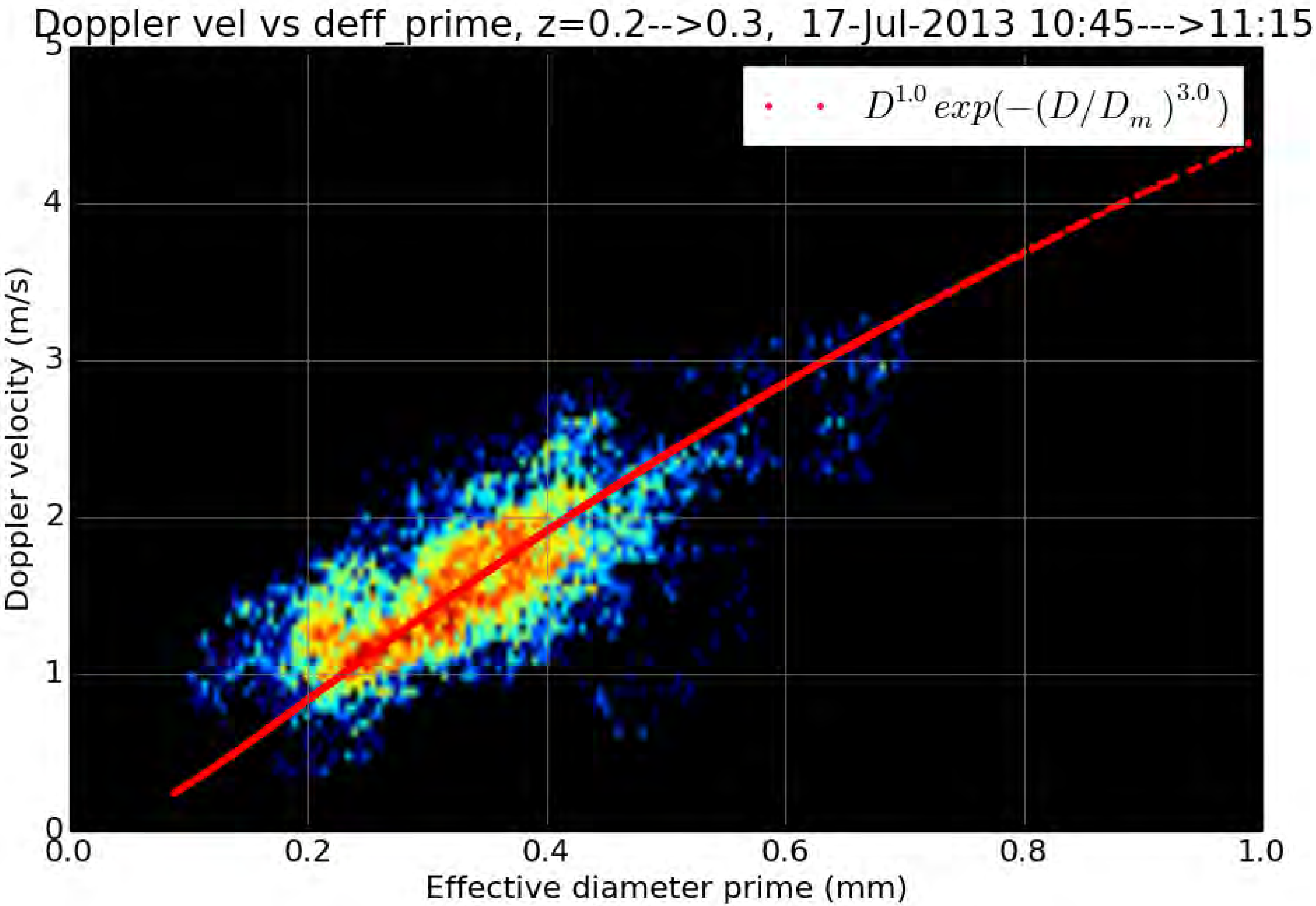


Spectral width vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha} exp(-(D/D_m)^{3.0})$
Spectral width (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0

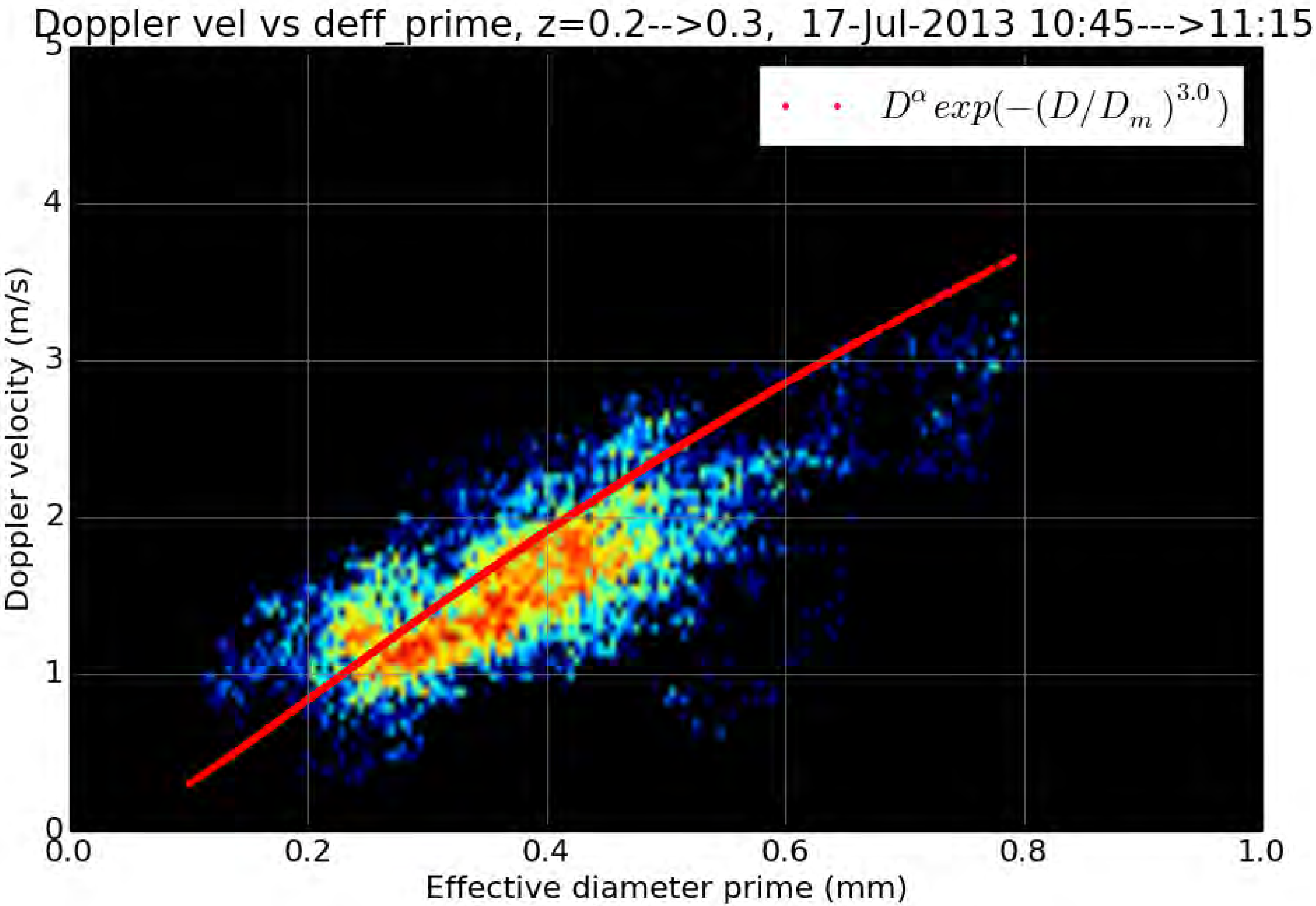

Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{1.0}exp(-(D/D_m)^{3.0})$
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0


Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha}\, exp(-(D/D_m)^{3.0})$
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0

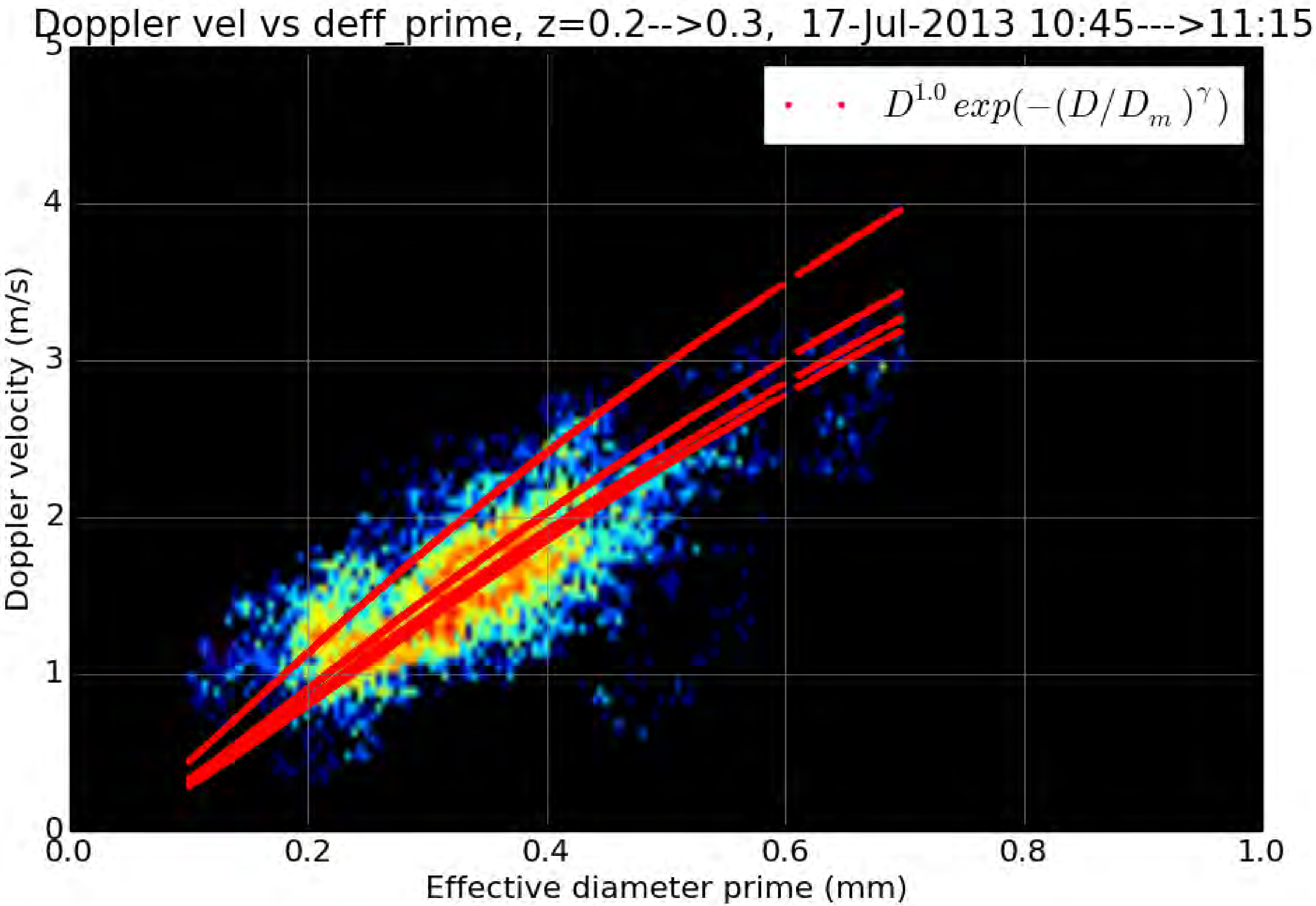

Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha} exp(-(D/D_m)^{3.0})$
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0

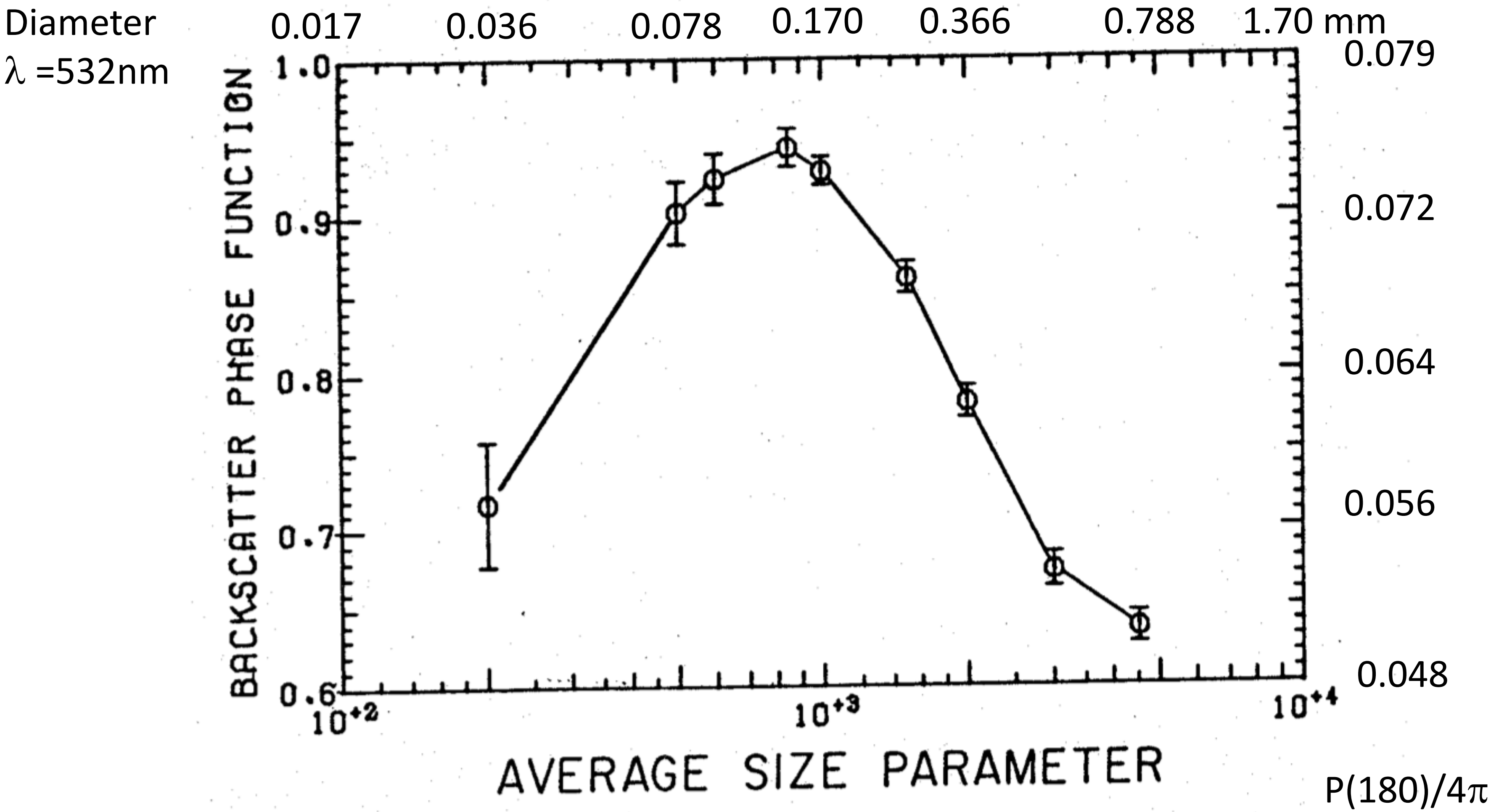


Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{1.0} exp(-(D/D_m)^{3.0})$
Doppler velocity (m/s)
Effective diameter prime (mm)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5



Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{\alpha} exp(-(D/D_m)^{3.0})$
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0


Doppler vel vs deff_prime, z=0.2-->0.3, 17-Jul-2013 10:45--->11:15
$D^{1.0} exp(-(D/D_m)^\gamma)$
Doppler velocity (m/s)
Effective diameter prime (mm)
0
1
2
3
4
5
0.0
0.2
0.4
0.6
0.8
1.0



Backscatter phase function for water drops averaged over size parameter intervals of 0.14. (Shipley 1978)