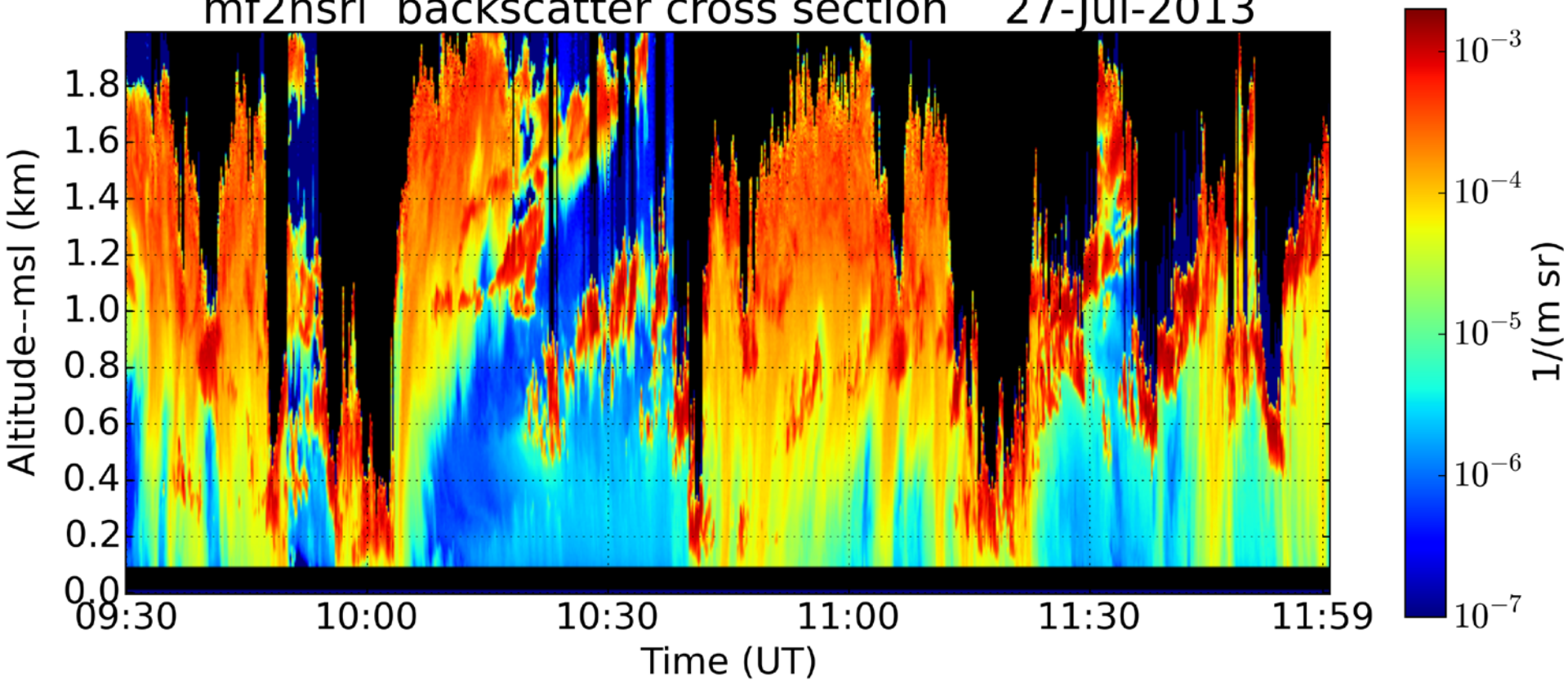
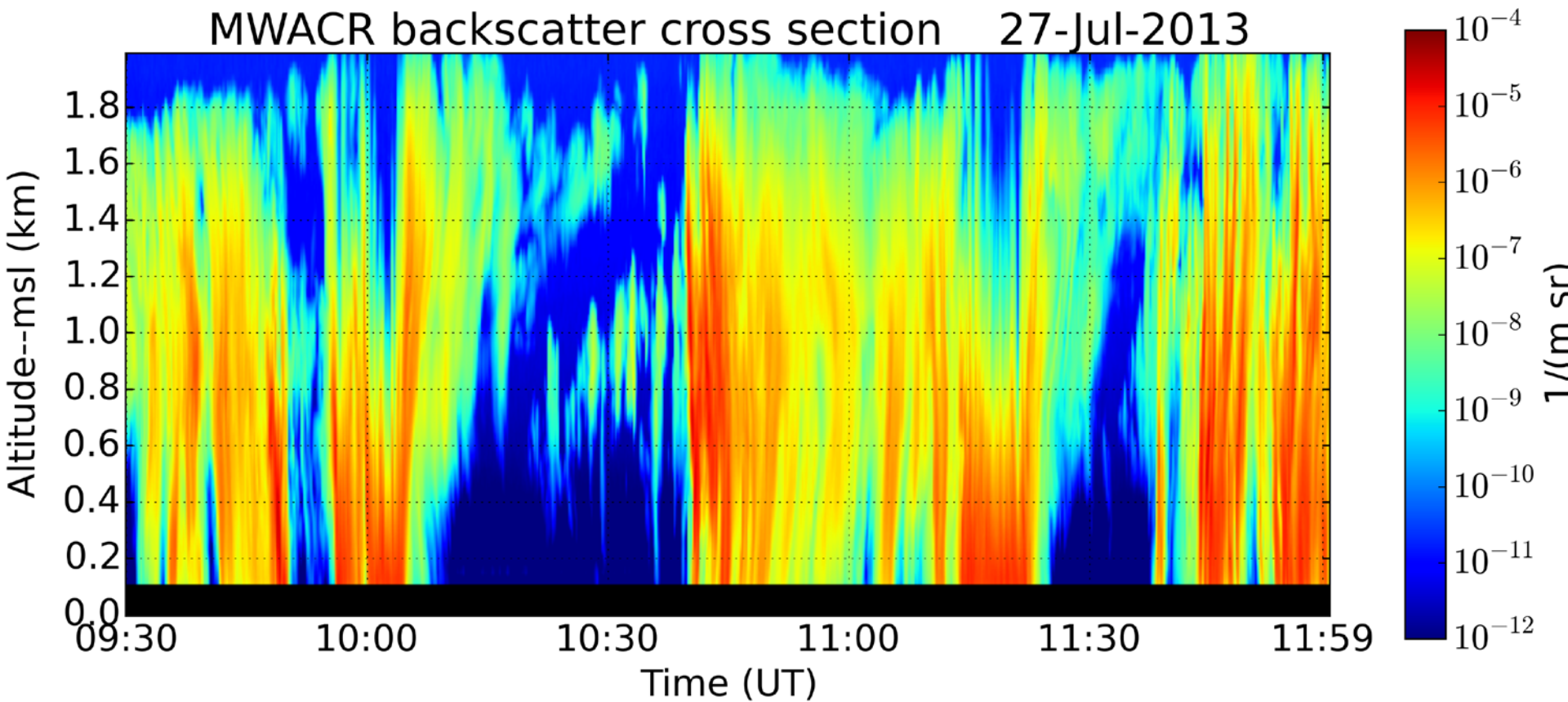


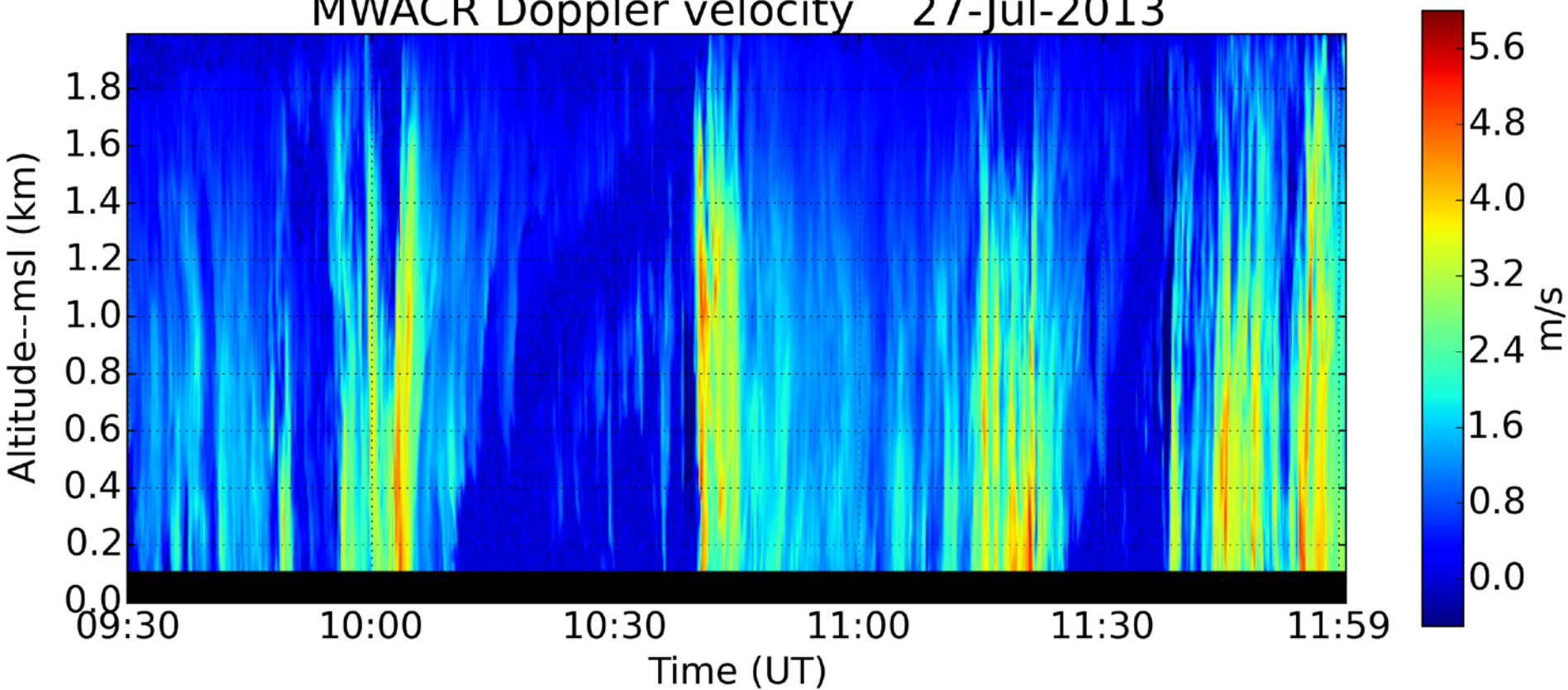
mf2hsrl backscatter cross section 27-Jul-2013



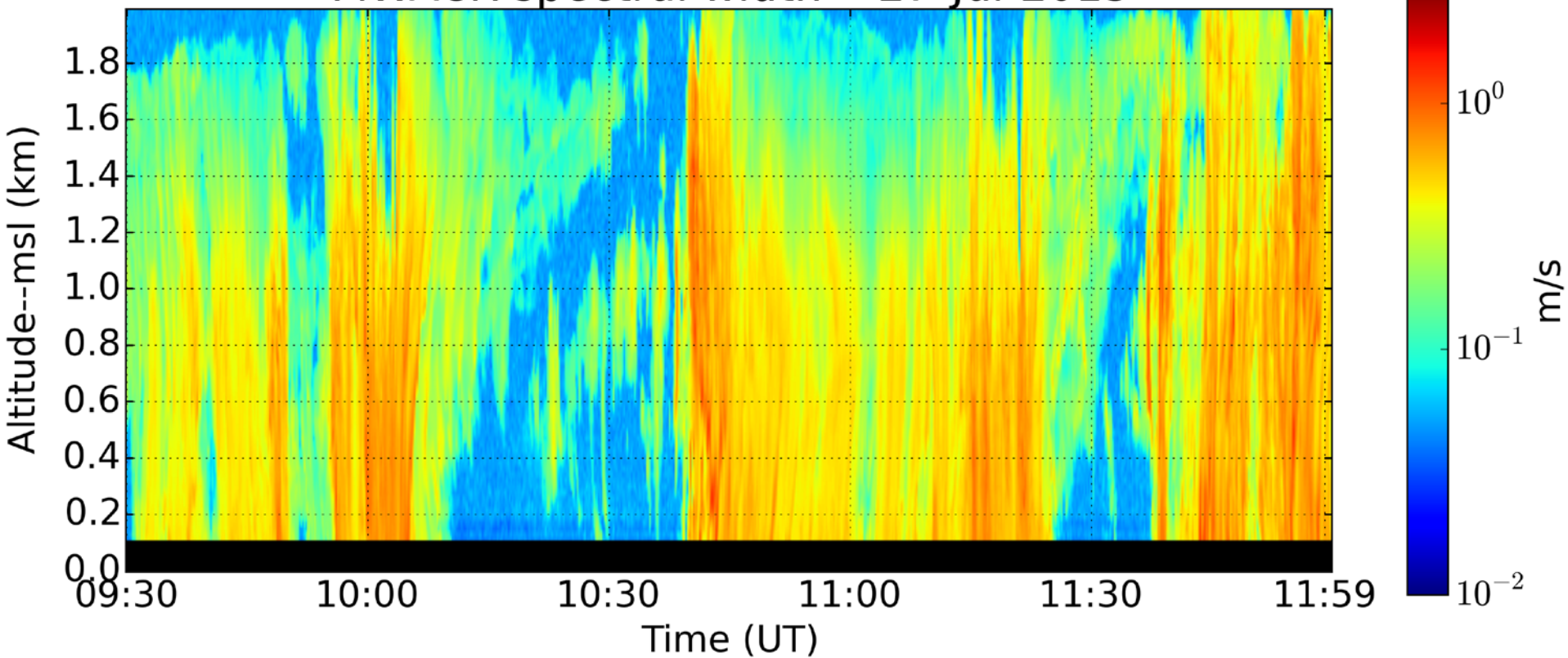
MWACR backscatter cross section 27-Jul-2013



MWACR Doppler velocity 27-Jul-2013



MWACR spectral width 27-Jul-2013



HSRL-Radar particle size measurement

For droplets which are small compared to the radar wavelength but large compared to the lidar wavelength:

Radar scattering cross section = $\beta_{\text{radar}} \sim \langle V^2 \rangle \sim \langle D^6 \rangle$ ----- Rayleigh scattering
Lidar extinction cross section = $\beta_{\text{lidar}} \sim \langle A \rangle \sim \langle D^2 \rangle$ ----- Geometric optics

Where:

$\langle V^2 \rangle$ = average volume squared of the particles

$\langle A \rangle$ = average projected area of the particles

We can define:

$$D'_{eff} = \lambda \cdot \left(\frac{2.0}{\pi^3 \cdot k_w^2} \right)^{0.25} \cdot \left(\frac{\beta_{bs\text{Radar}}}{\beta_{ext\text{Lidar}}} \right)^{0.25}$$

Where:

λ_{radar} = radar wavelength

k_w = dielectric constant of water

Where: $\beta_{bs\text{Radar}} \left(\frac{1}{m \cdot sr} \right) = \frac{3}{8\pi} \left(\frac{1}{sr} \right) \cdot \beta_{scat} \left(\frac{1}{m} \right)$

The extinction cross section, β_e , can be derived from the molecular return by taking the logarithm and then differentiating with respect to range, r :

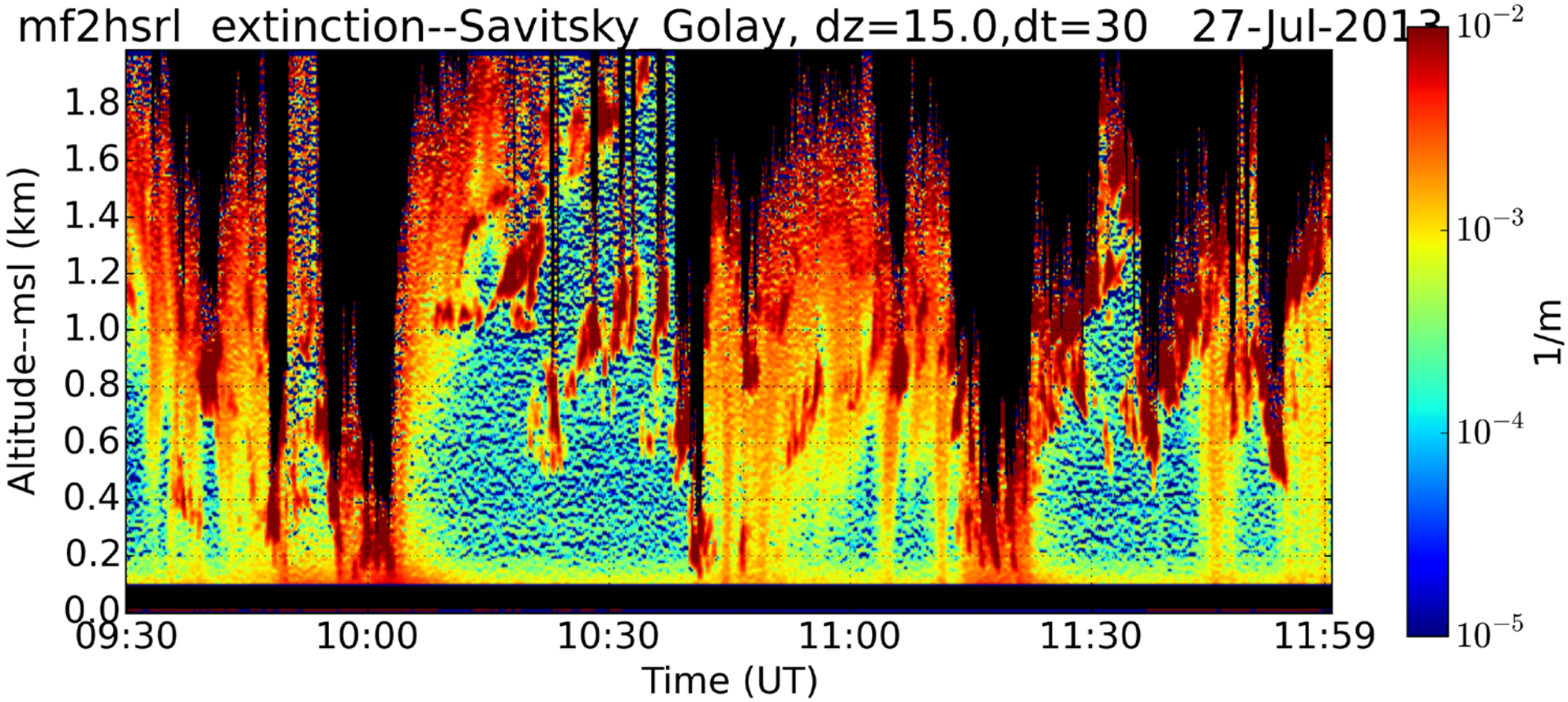
$$S_m(r) \sim \eta(r) \cdot \frac{1}{r^2} \cdot \frac{3}{8\pi} \cdot \beta_m(r) \cdot e^{-2 \int_0^r \beta_e(r) dr}$$

Taking the logarithm and differentiating with respect to range
Provides the extinction cross section in terms of S_m and β_m :

$$\beta_e(r) = \frac{1}{2} \cdot \frac{d}{dr} \left[\log \left(\frac{\eta(r) \cdot \beta_m(r)}{r^2 \cdot S_m(r)} \right) \right]$$

Unfortunately, the geometric correction, $\eta(r)$, does not cancel out:

mf2hsrl extinction--Savitsky Golay, dz=15.0,dt=30 27-Jul-2012

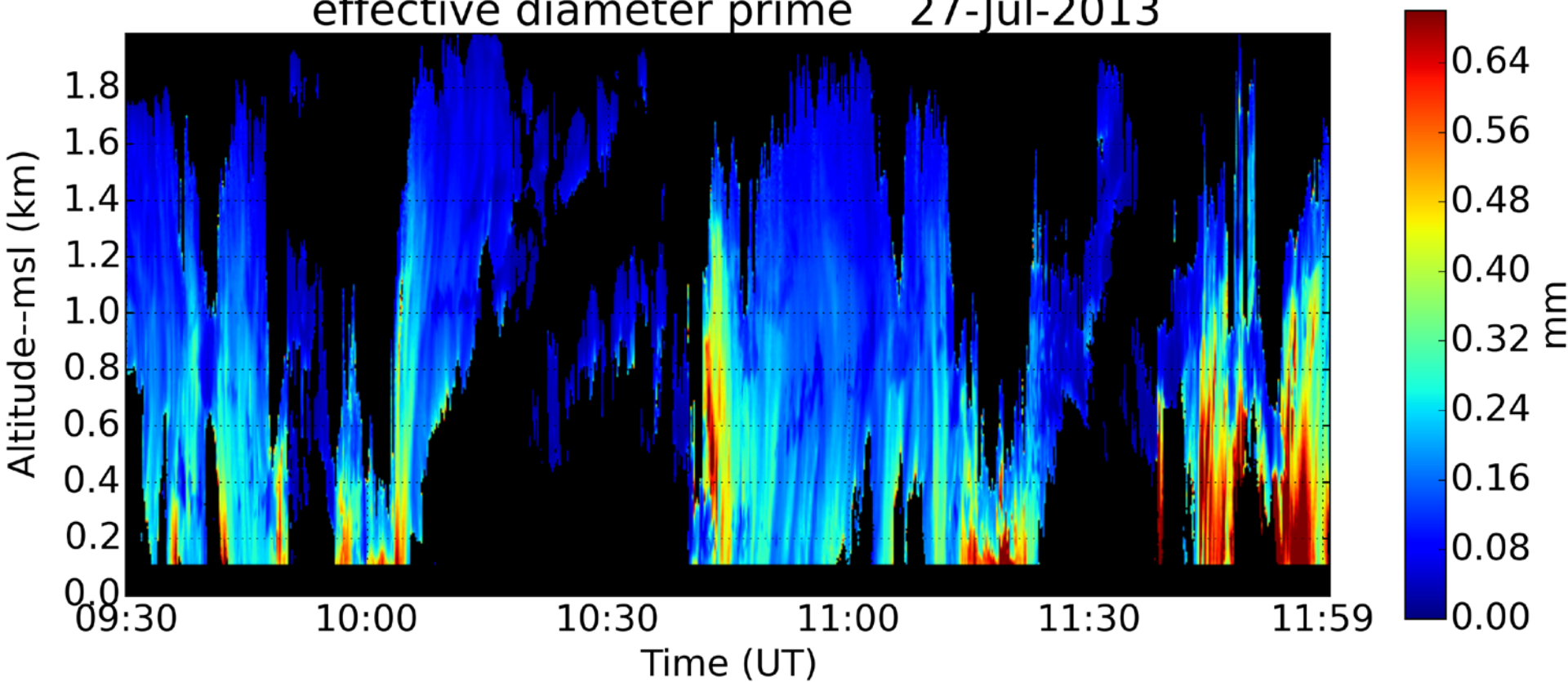


The HSRL provides robustly calibrated retrievals of the backscatter cross section. Thus given a value for the backscatter phase function we can derive the extinction cross section from:

$$\beta_{ext} = \beta_{bs} \cdot \frac{4\pi}{P(180)}$$

In the past we used a constant value, $\frac{P(180)}{4\pi} = 0.065$, derived from direct multiple scattering corrected HSRL extinction measurements in drizzle.

effective diameter prime 27-Jul-2013



Assuming a gamma distribution of particle diameters:

$$N = a \int_0^{\infty} D^{\alpha} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \left(\frac{D}{D_m}\right)^{\gamma}\right) \cdot dD$$

Where:

N = number of particles

D = particle Diameter

D_m = mode diameter

The effective diameter prime can be written as:

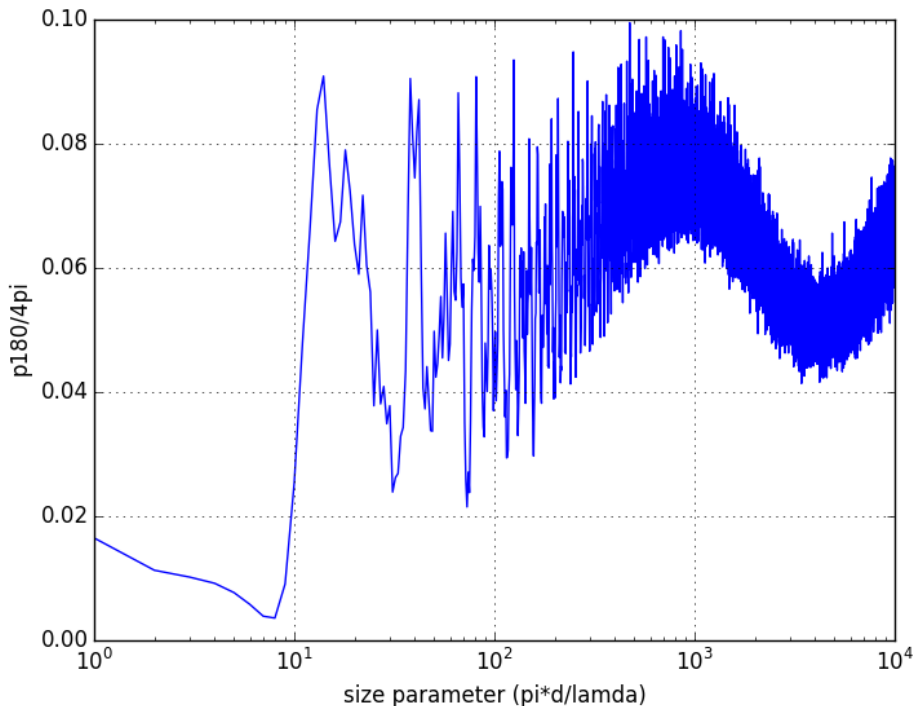
$$D'_{eff} = \frac{\int D^{\alpha+6} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^{\gamma}\right) \cdot dD}{\int D^{\alpha+2} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^{\gamma}\right) \cdot dD} = \frac{\frac{\alpha}{\gamma}^{\frac{1}{\gamma}} \cdot D_m \cdot \Gamma\left(\frac{\alpha+7}{\gamma}\right)}{\Gamma\left(\frac{\alpha+3}{\gamma}\right)}$$

Solving for the mode diameter, D_m :

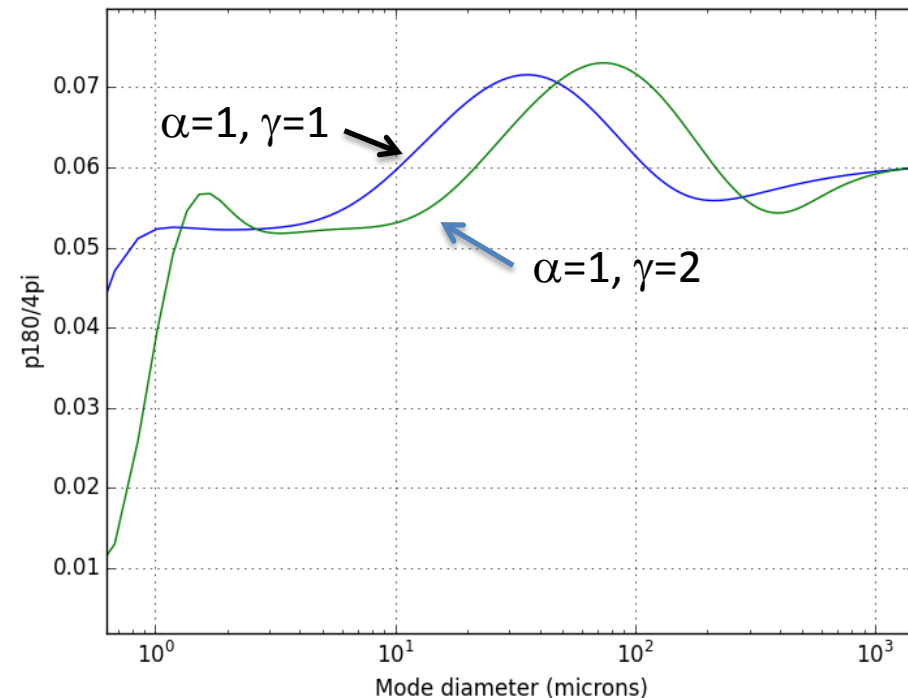
$$D_m = D'_{eff} \cdot \left(\frac{\alpha}{\gamma}\right)^{\frac{1}{\gamma}} \cdot \left(\frac{\Gamma\left(\frac{\alpha+3}{\gamma}\right)}{\Gamma\left(\frac{\alpha+7}{\gamma}\right)}\right)^{\frac{1}{4}}$$

Backscatter phase function for drizzle from Mie theory

Backscatter phase function vs size parameter averaged over $\delta x = 1$ intervals.



Backscatter phase function vs mode diameter after smoothing over gamma distribution of particles sizes, $\lambda = 532\text{nm}$.

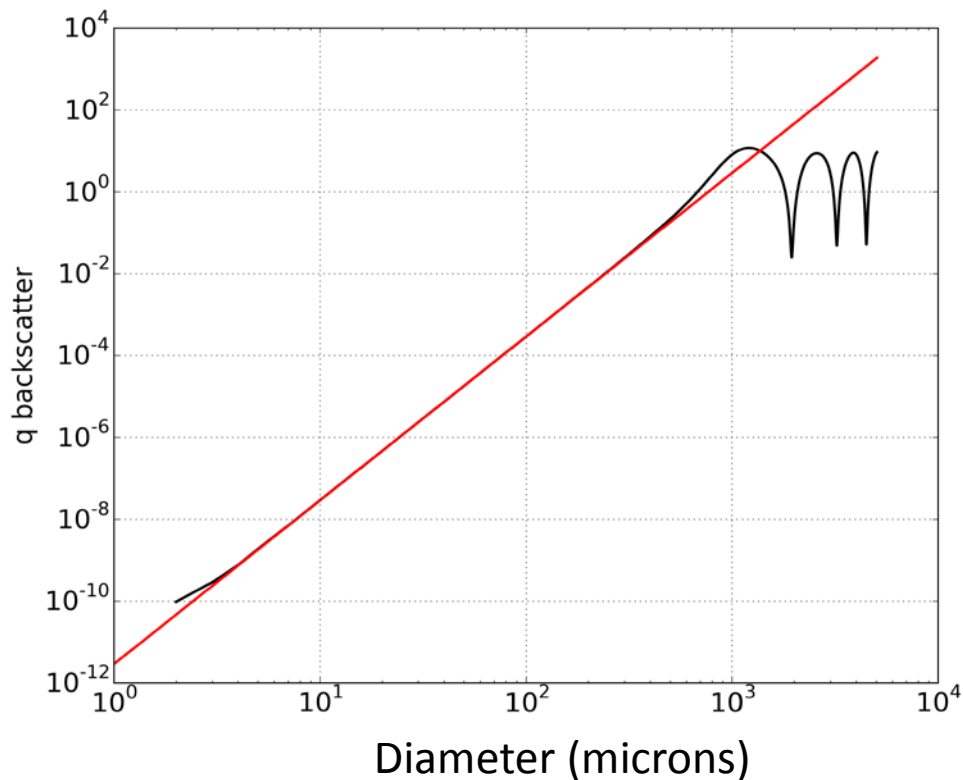


Gamma distribution:

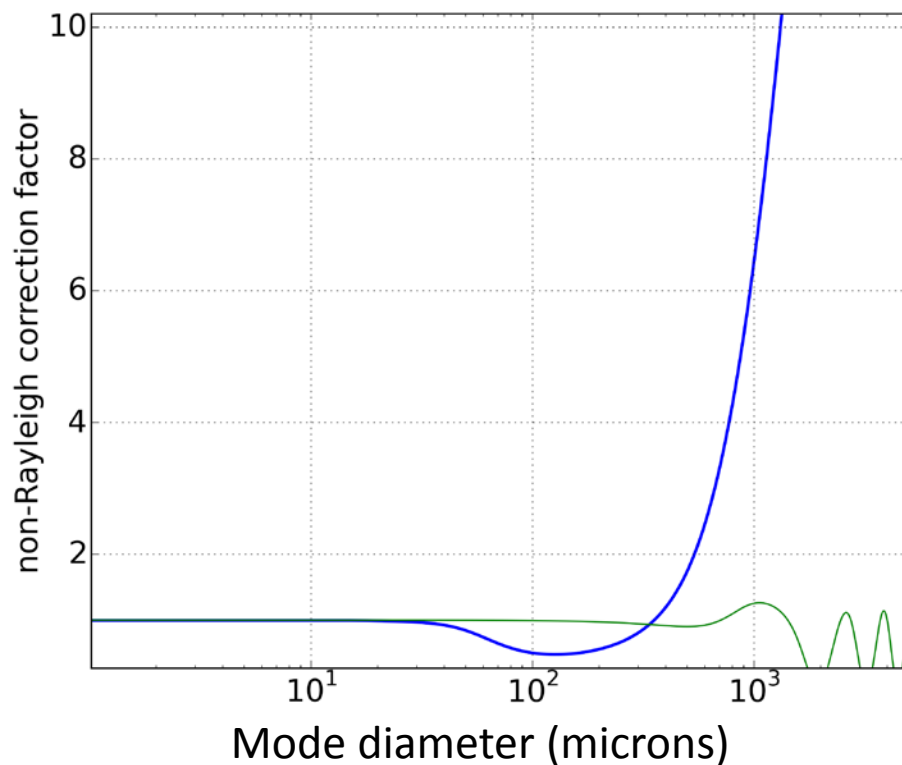
$$N = a \int_0^{\infty} D^{\alpha} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \left(\frac{D}{D_m}\right)^{\gamma}\right) \cdot dD$$

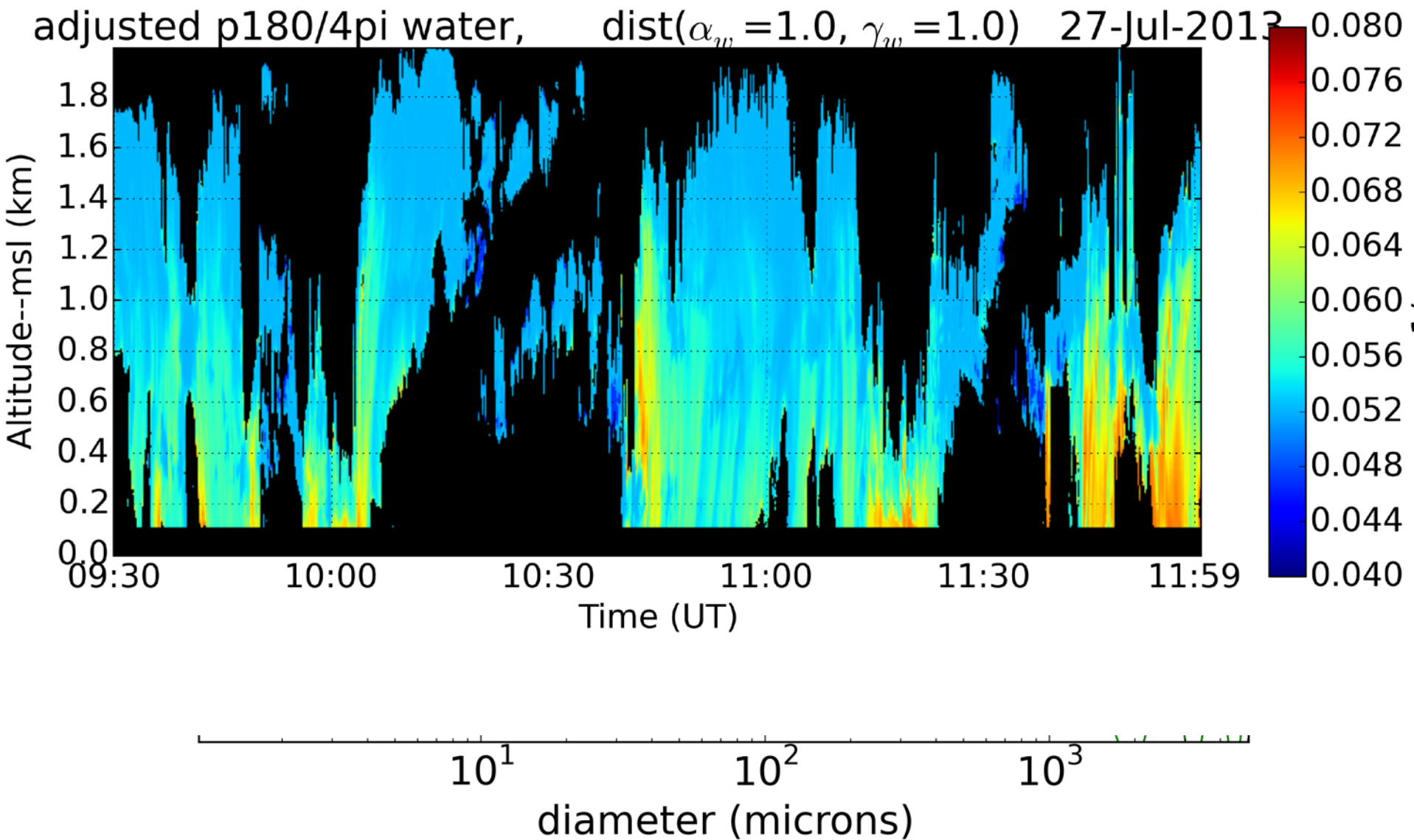
Non-Rayleigh correction to radar backscatter

Backscatter efficiency for mono-disperse particle distribution at $\lambda = 3.2$ mm.

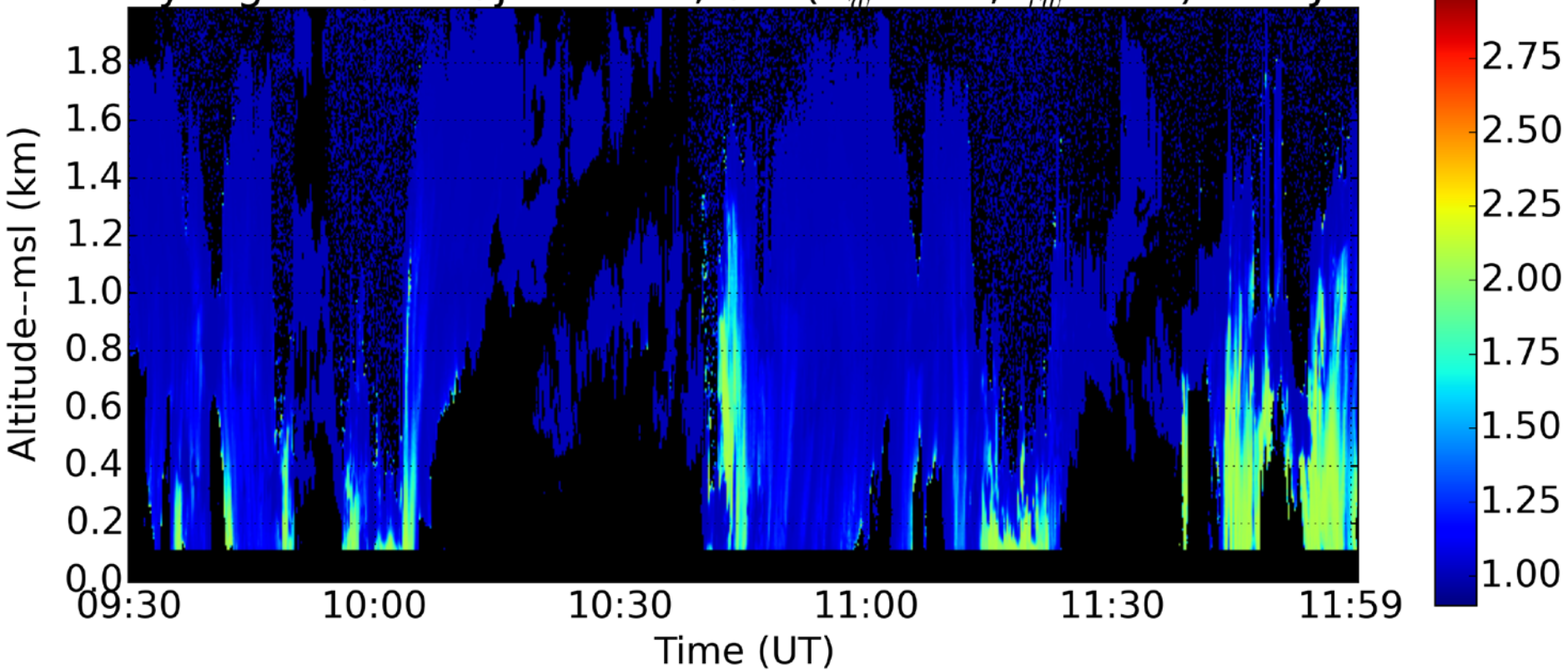


Non-Rayleigh correction factor for gamma distribution of particle sizes $\alpha=1, \gamma=1$ (bl), $P(180)/4\pi * 8\pi/3$ (grn)





Non-Rayleigh radar adjustment, $\text{dist}(\alpha_w = 1.0, \gamma_w = 1.0)$ 27-Jul-2013 00:00



The radar weighted fall velocity, $\langle V_{rf} \rangle$:

$$\langle V_{rf} \rangle = \frac{\int V_f \cdot D^{\alpha+6} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^\gamma\right) \cdot dD}{\int D^{\alpha+6} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^\gamma\right) \cdot dD}$$

And the mass weighted fall velocity, $\langle V_{mf} \rangle$:

$$\langle V_{mf} \rangle = \frac{\int V_f \cdot D^{\alpha+3} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^\gamma\right) \cdot dD}{\int D^{\alpha+3} \cdot \exp\left(-\frac{\alpha}{\gamma} \cdot \frac{D}{D_m}^\gamma\right) \cdot dD}$$

Where the fall velocity, V_f , is computed from:

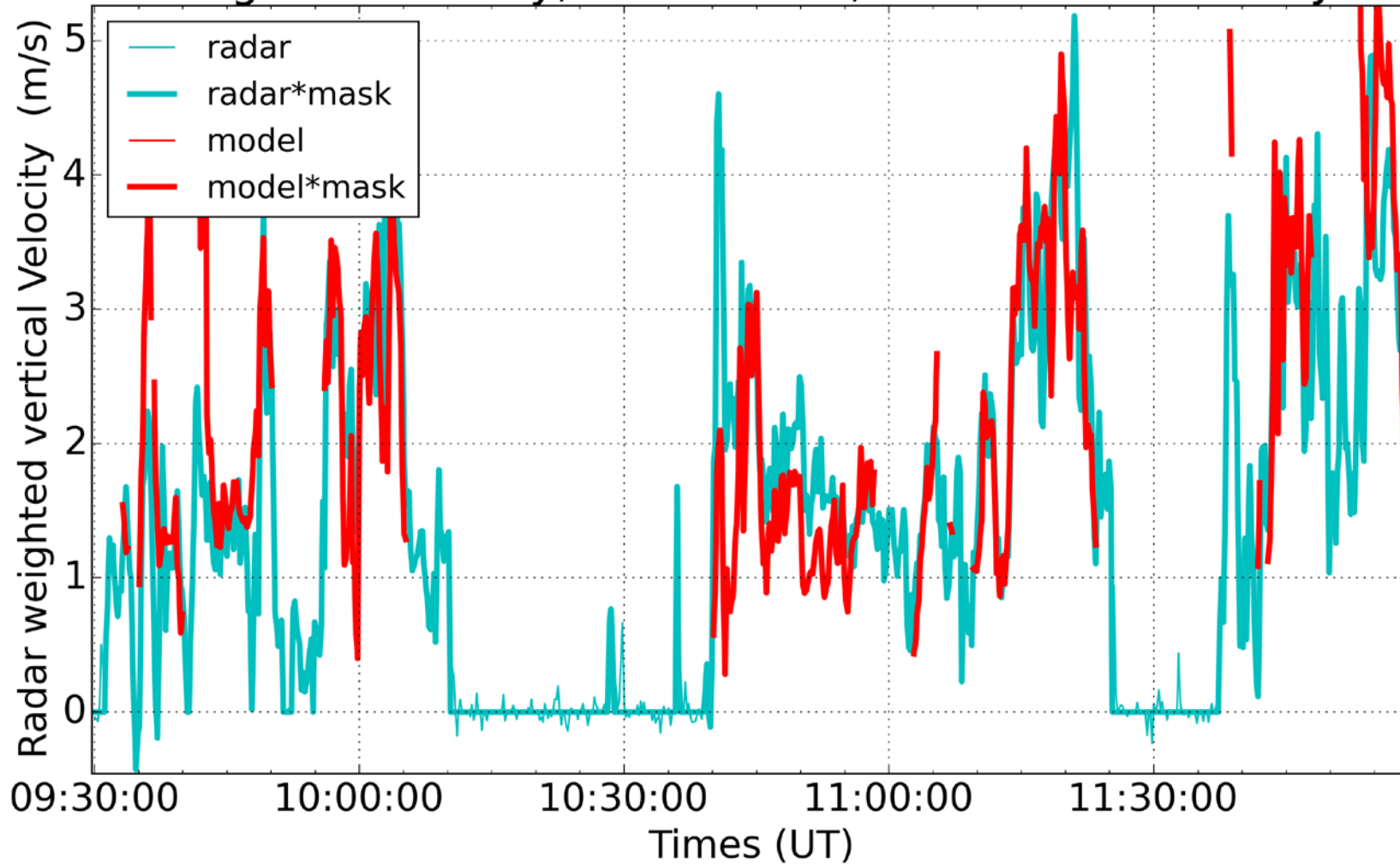
$$V_f = \frac{\eta}{\rho_{air} D} \left(\frac{\delta_0^2}{4} \left[\left(1 + C_1 X^{\frac{1}{2}}\right)^{\frac{1}{2}} - 1 \right]^2 - a_0 X^{b_0} \right)$$

Khovostyanov and Curry, JAS May 2005, Vol 62

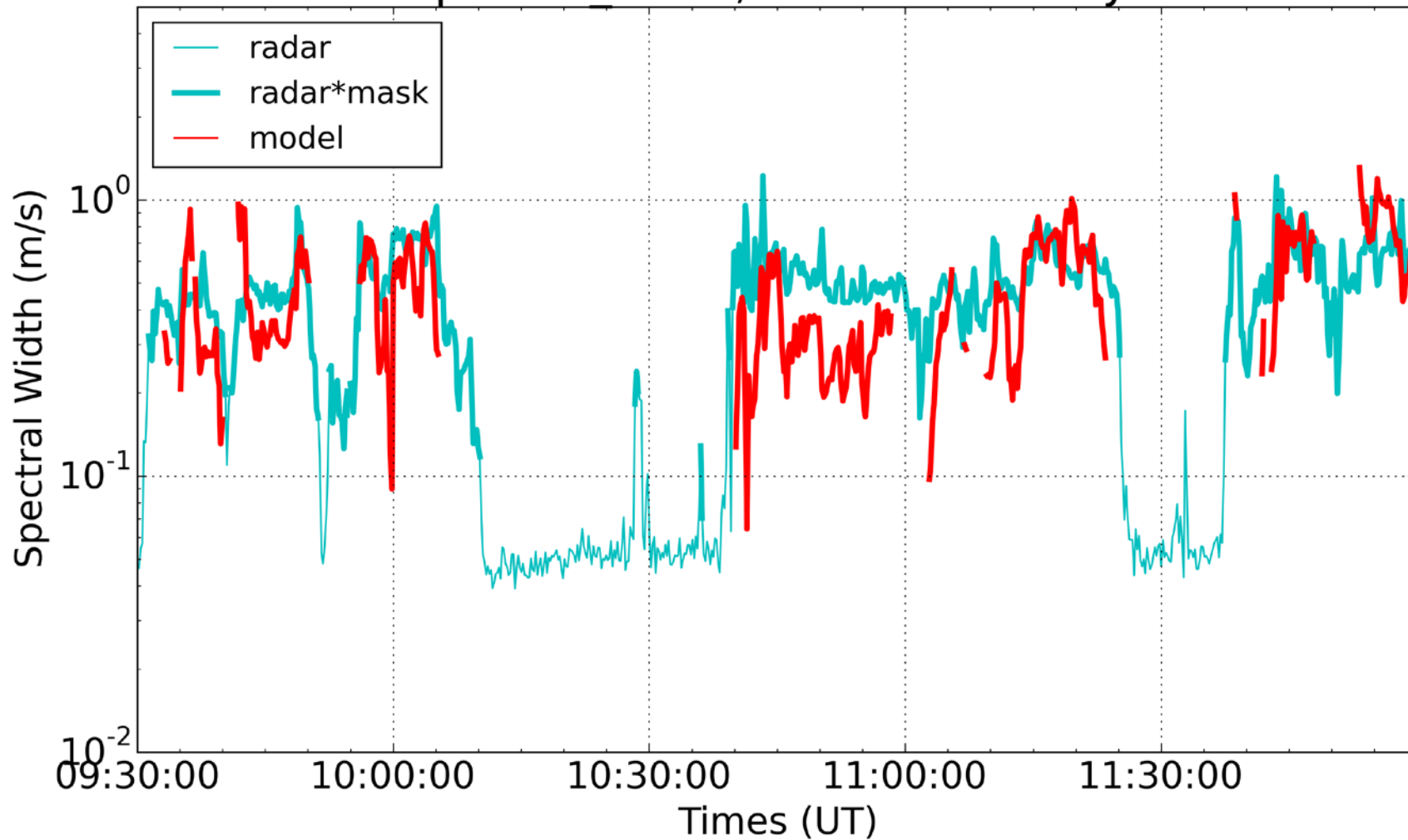
And the Beard Number, X expressed in terms of particle area, volume and density along with the acceleration of gravity, air density, particle diameter and the dynamic viscosity:

$$X = \frac{2 \cdot \text{volume} \cdot \rho_{particle} \cdot \rho_{air} \cdot g \cdot D^2}{\text{area} \cdot \eta^2}$$

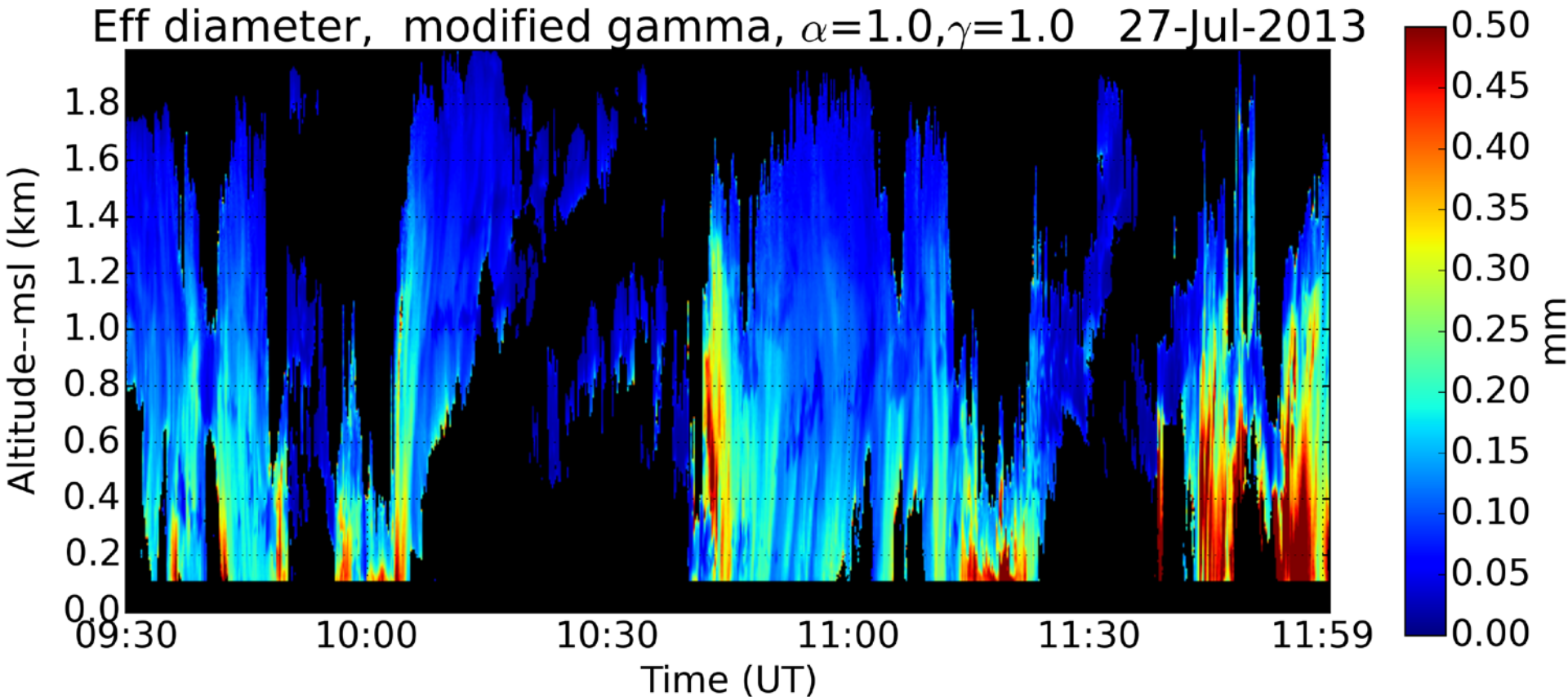
radar weighted velocity, $z = 0.13$ km, $\langle dv \rangle = 0.867$ 27-Jul-2013



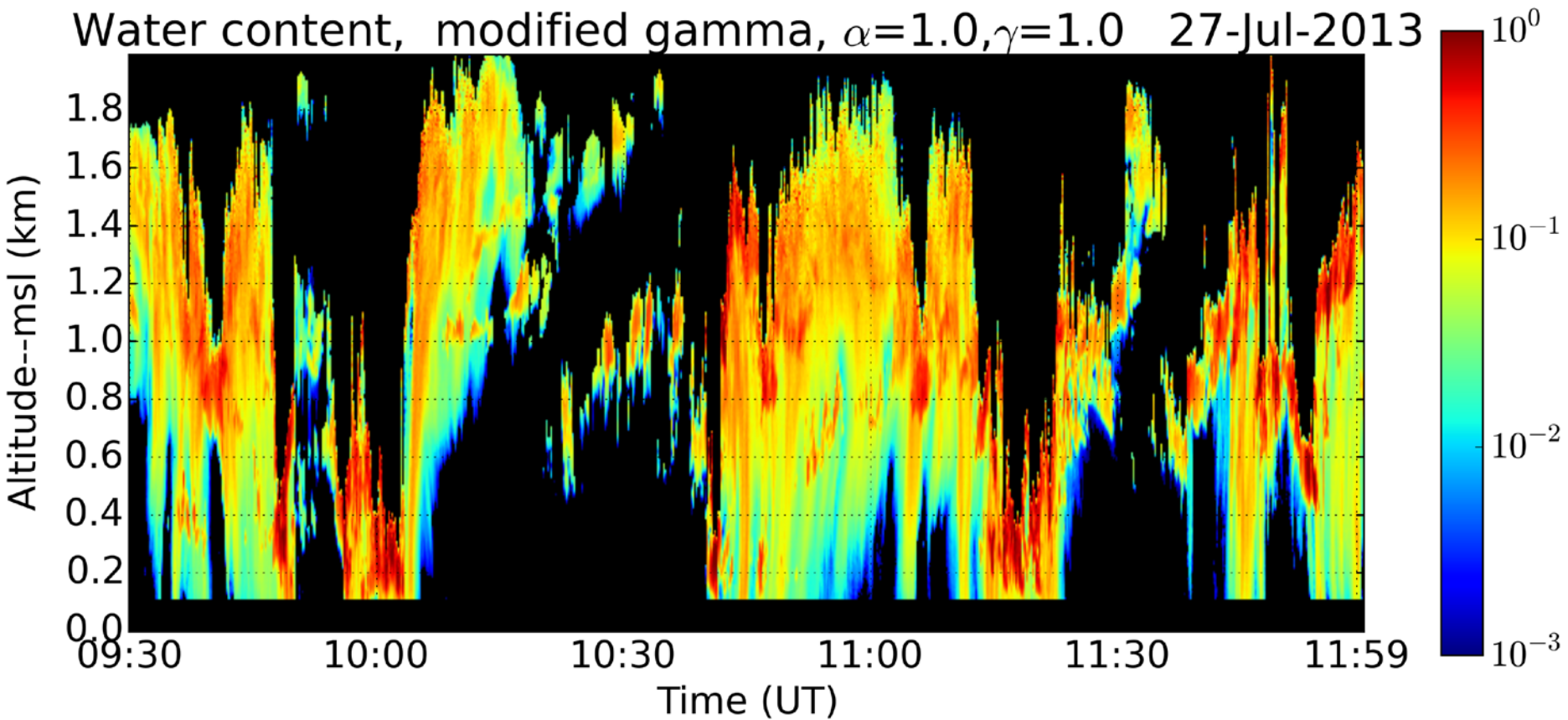
Radar spectral width, z=0.13 km 27-Jul-2013



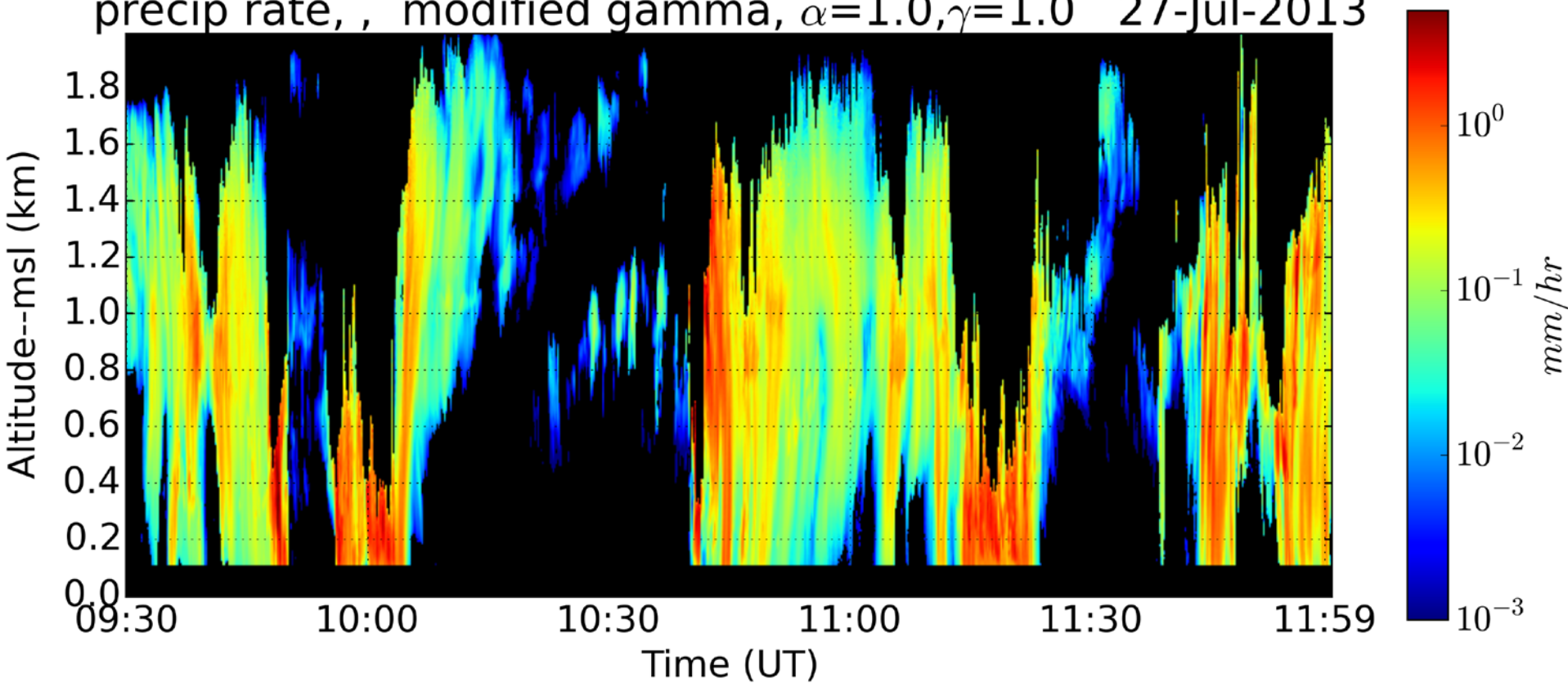
Eff diameter, modified gamma, $\alpha=1.0, \gamma=1.0$ 27-Jul-2013



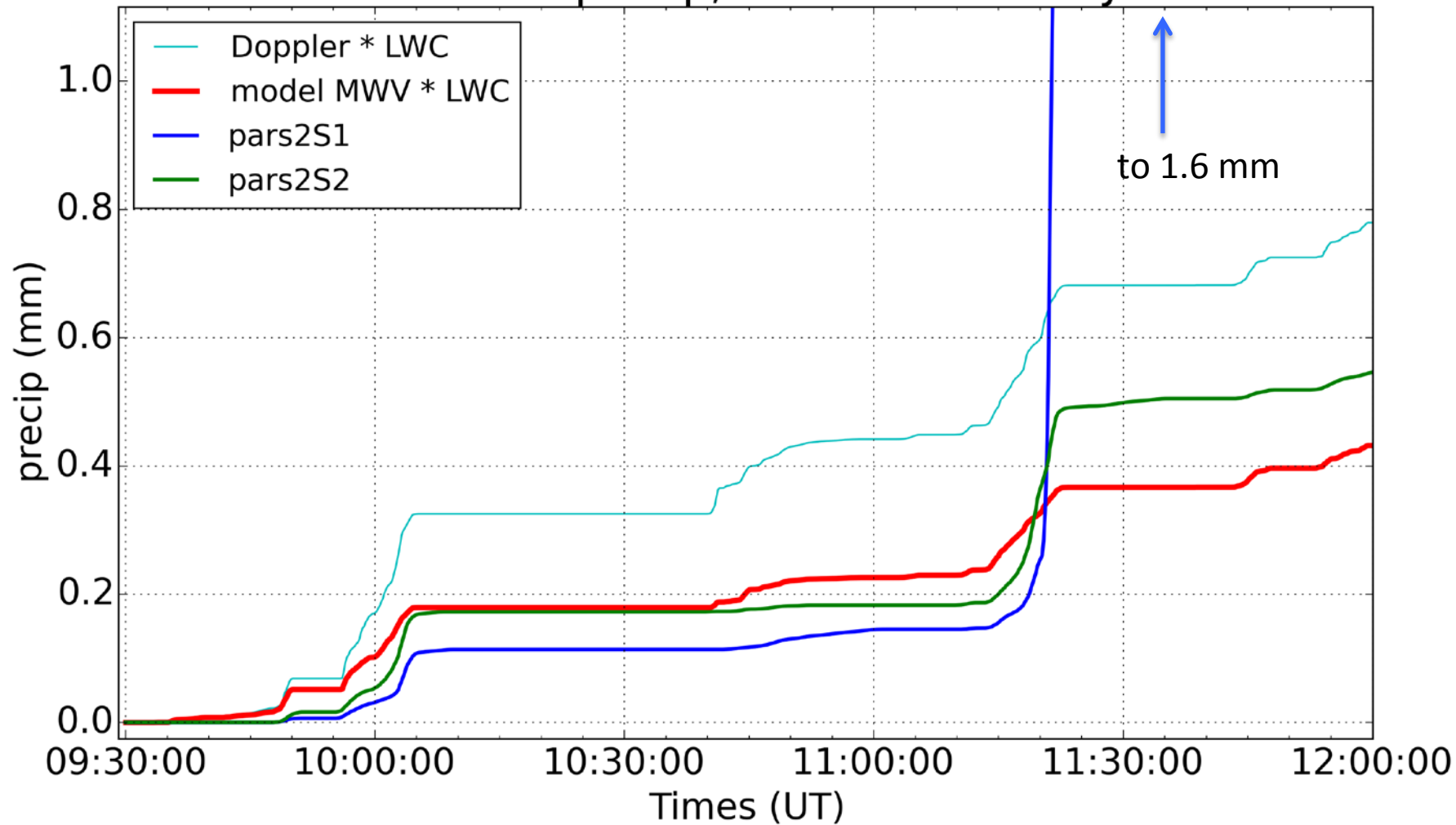
Water content, modified gamma, $\alpha=1.0, \gamma=1.0$ 27-Jul-2013



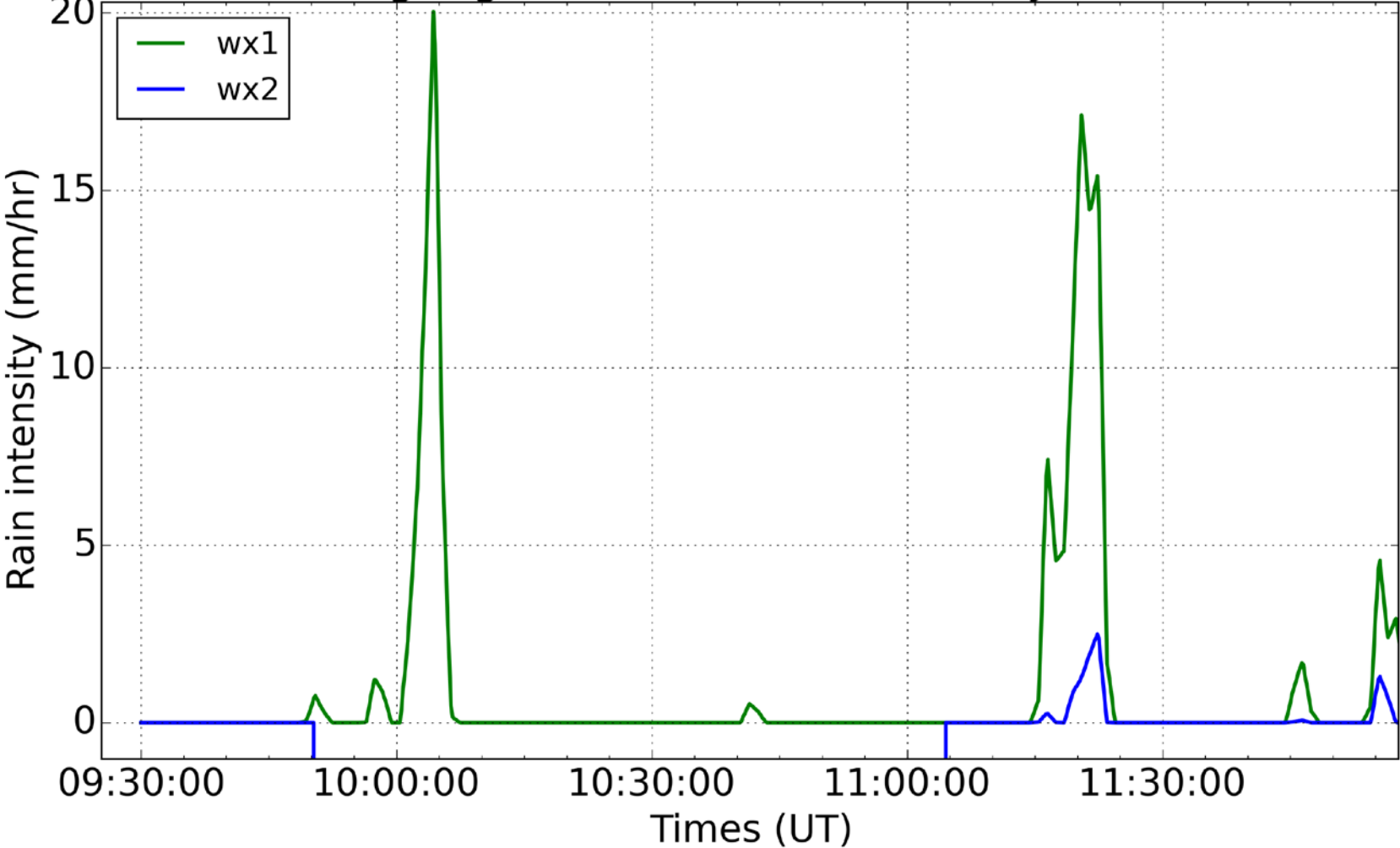
precip rate, , modified gamma, $\alpha=1.0, \gamma=1.0$ 27-Jul-2013



mf2hsrl-MWACR precip, z = 0.13 km 27-Jul-2013

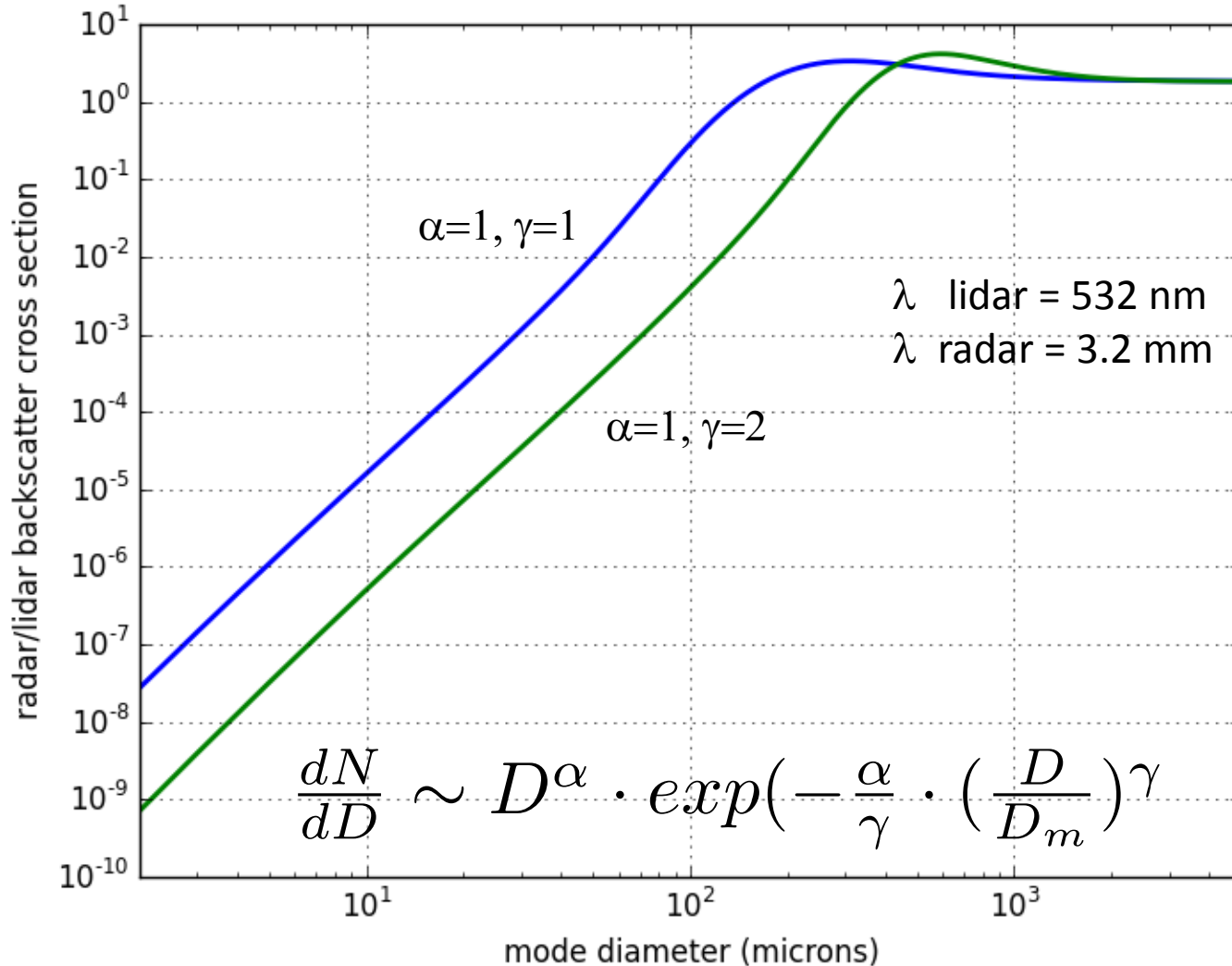


rain gauge Rain rate vs time 27-Jul-2013

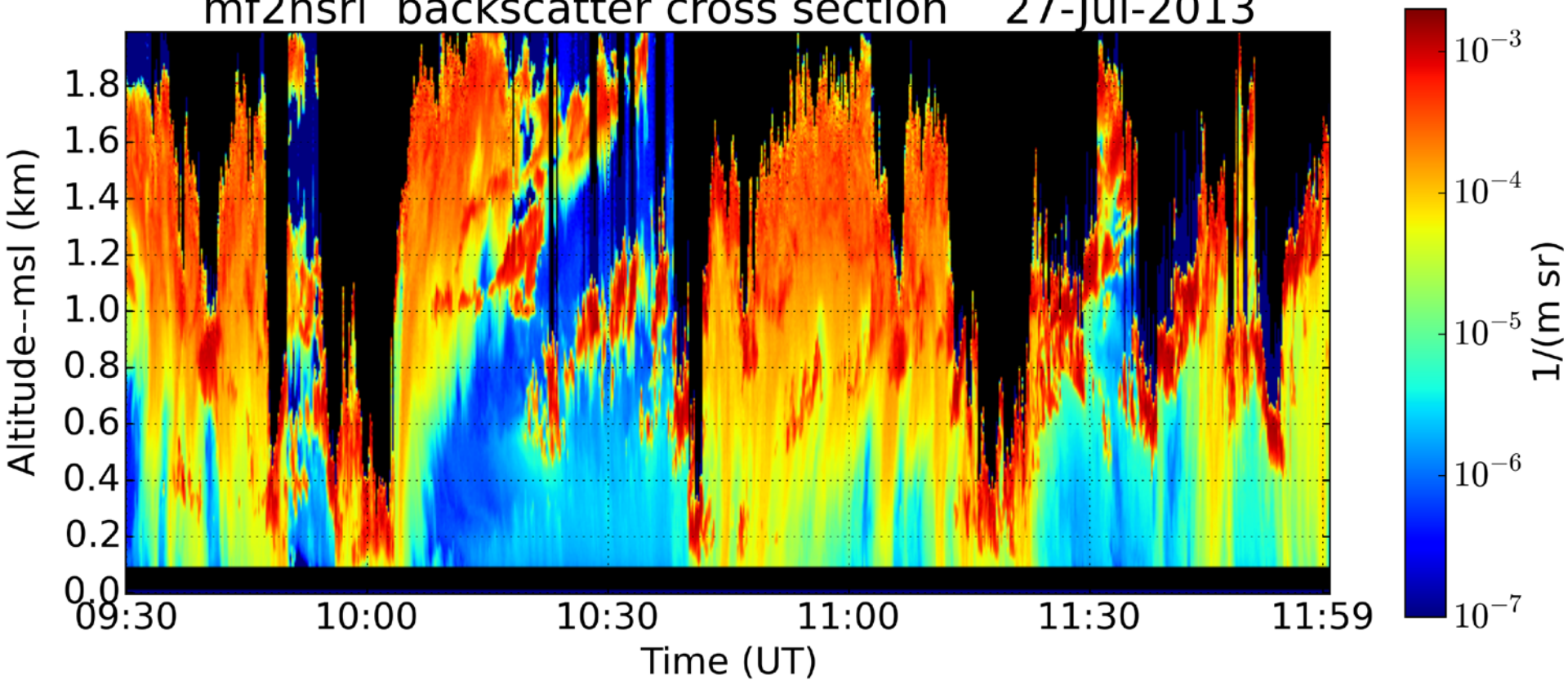


$\frac{\beta_{bsRadar}}{\beta_{bsLidar}}$

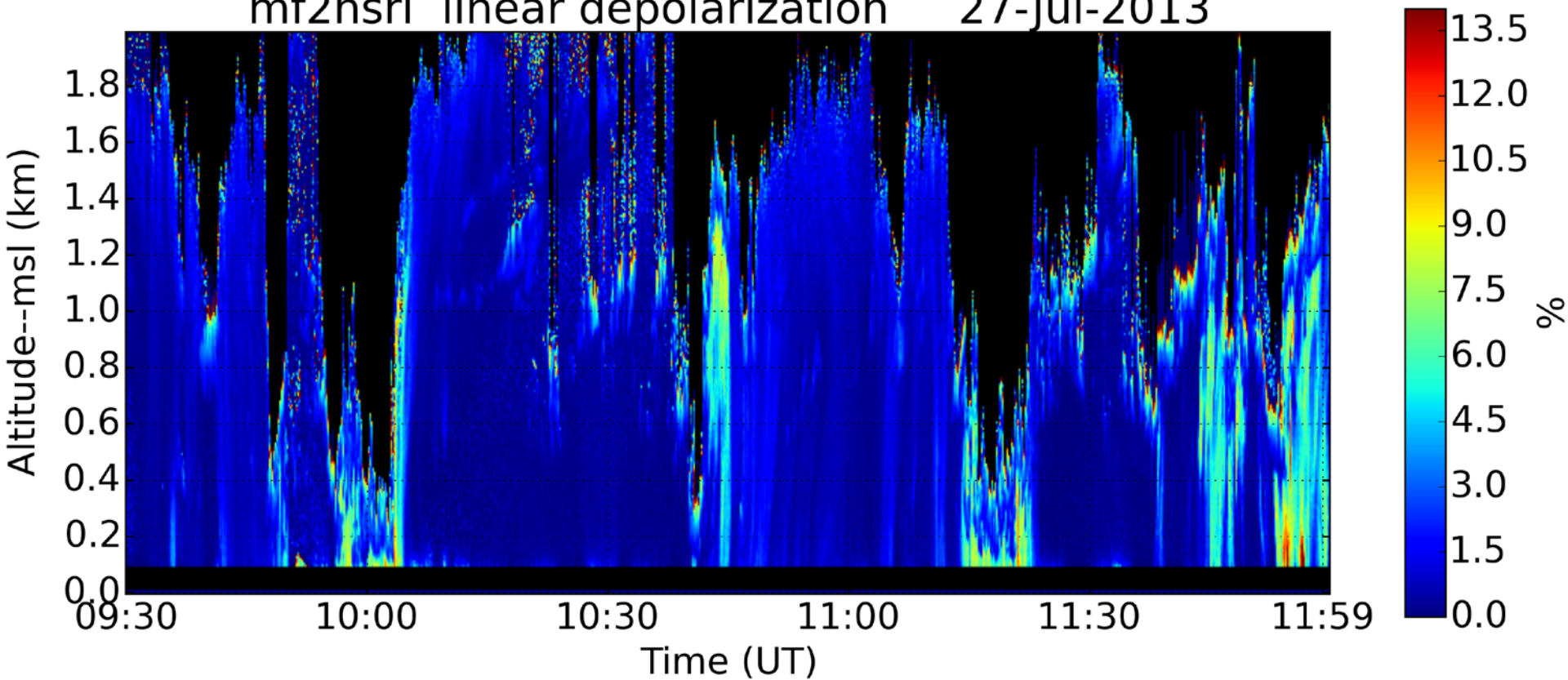
vs D_{mode} from Mie theory using gamma distribution



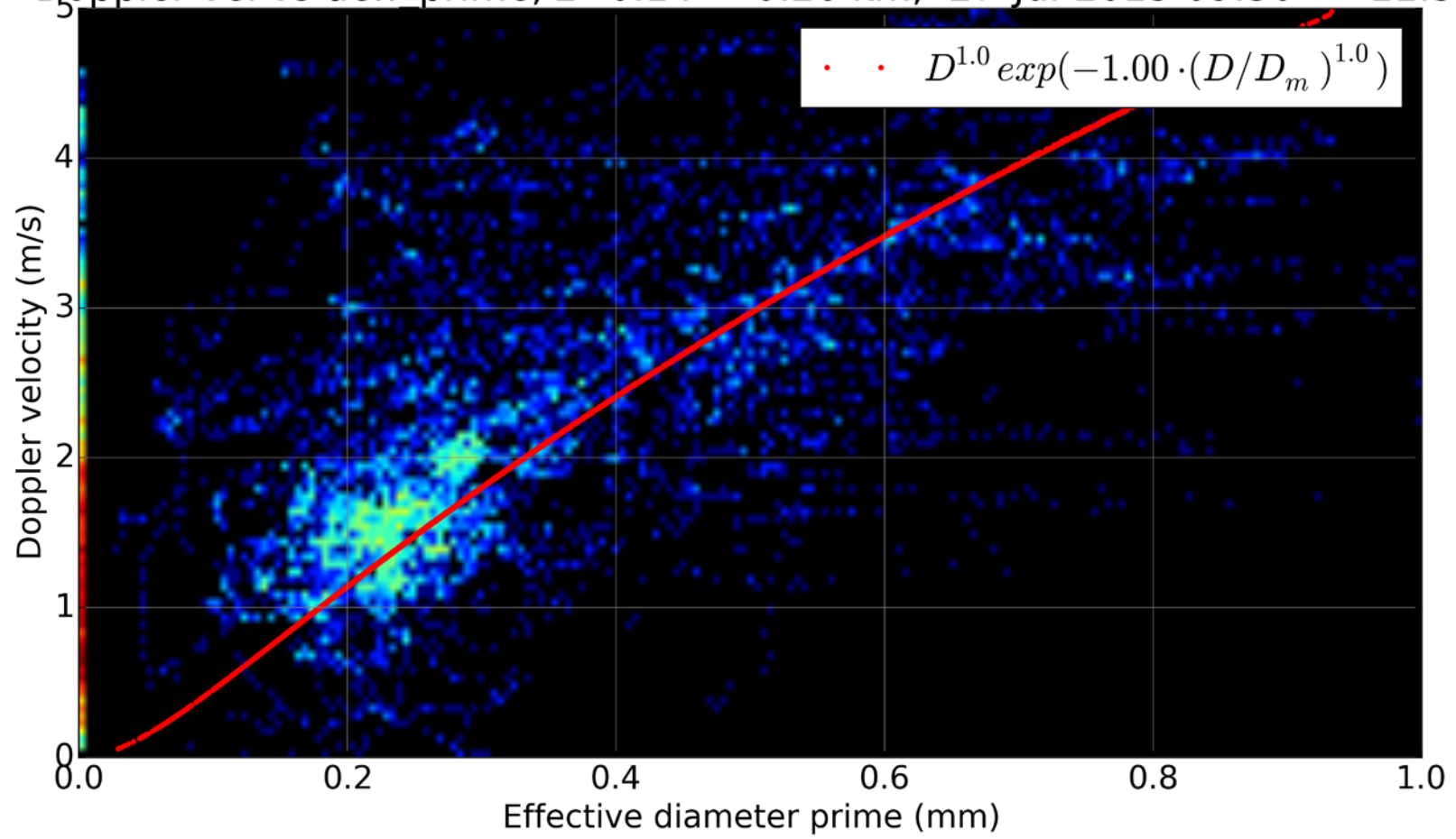
mf2hsrl backscatter cross section 27-Jul-2013



mf2hsrl linear depolarization 27-Jul-2013



Doppler vel vs deff prime, z=0.14-->0.26 km, 27-Jul-2013 09:30--->11:59



Spectral width vs deff prime, z=0.14-->0.26 km, 27-Jul-2013 09:30--->11:59

