Potential of Higher Moments of the Radar Doppler Spectrum for Studying Ice Clouds

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Vertically pointing Doppler radar

- Give profile information
- High spatiotemporal resolution
- But: Indirect measurement & high uncertainties!

Motivation: We have to increase the number of observables!
What does a Doppler radar measure?

- Reflectivity $Ze$ is sensitive to false radar calibration
- Mean Doppler Velocity $W$ is sensitive to vertical air motion
Higher order moments

- In addition to $Z_e$ and $W$, use also:
  - Spectrum Width $\sigma$
Higher order moments

- In addition to $Ze$ and $W$, use also:
  - Spectrum Width $\sigma$
  - Skewness $Sk$
  - Kurtosis $Ku$
  - Left and Right Slope $S_{l,r}$

- Strongly influenced by kinematic broadening $\sigma_k$ (turbulence, horizontal wind)

What additional information can be provided by higher moments?
Degrees of freedom for signal

- Estimate the number of independent information pieces
- Use Bayesian retrieval (Optimal Estimation)
  1. Develop forward operator -> PAMTRA
  2. Get a priori data set -> ISDAC + Parameterizations
  3. Apply Retrieval
Ingredients to simulate a Doppler spectrum with PAMTRA

Required variables:

- Normalized Gamma distribution $N(D)$: 3 parameters
- Mass-size relation $m(D)$: 2 p.
- Cross section area $A(D)$: 2 p.
- Vertical wind $w$: 1 p.
- Turbulent broadening $\sigma_k$: 1 p.

- 9(!) state variables
A priori data set

- Indirect and Semi-Direct Aerosol Campaign (ISDAC)
- *low to medium* turbulence Stratocumulus ice clouds
- April 2008 in Alaska
- Convair 580 with in situ instruments
Degrees of freedom for 1300 profiles

- For $K_a$-band observations, higher moments can double the information content.
- Using all moments provides more information for one frequency than lower moments for two frequencies.
Degrees of freedom for 1300 profiles

- For $K_a$-band observations, higher moments can double the information content.
- Using all moments provides more information for one frequency than lower moments for two frequencies.
- Results for real MMCR observations from Barrow agree well.
Relative posterior uncertainty [%]
Particle size distribution $\log_{10}(N_0)$

- Single-frequency lower moments retrieval cannot retrieve $N_0$ well
- Single-frequency *all* moments retrieval reduces $N_0$ uncertainty
- Dual/triple-frequency *all* moments retrieval reduces uncertainty by 50%
- Using only higher moments gives less enhancement

R: real measurements
1: single frequency
2: dual frequency
3: triple frequency
Enhancement for most quantities when using all moments and one frequency

R: real measurements
1: single frequency
2: dual frequency
3: triple frequency
Best results for all quantities when using all moments and two or three frequencies

R: real measurements
1: single frequency
2: dual frequency
3: triple frequency
What happens if you have independent information about vertical air motion and turbulence? e.g. from other retrievals?

Improvements for quantities describing $m(D)$ and $A(D)$

- R: real measurements
- 1: single frequency
- 2: dual frequency
- 3: triple frequency
<table>
<thead>
<tr>
<th>Ze</th>
<th>W</th>
<th>σ</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Left slope</th>
<th>Right slope</th>
<th>Phase</th>
<th>Dm</th>
<th>N0*</th>
<th>μ*</th>
<th>m(D)</th>
<th>A(D)</th>
<th>A(D)</th>
<th>A(D)</th>
<th>A(D)</th>
<th>Kin</th>
<th>σk</th>
<th>Vertical wind</th>
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**ISDAC April, 8 2008**

**MMCR observation**

**Retrieval Result**
Summary

• Higher moments of the radar Doppler spectrum can be included into retrievals of polar arctic ice clouds
• Retrievals using also higher moments can enhance retrievals of numerous microphysical and kinematic quantities
  – Vertical air motion
  – Kinematic broadening (→ turbulence)
  – Particle distribution
  – Particle cross section Area
  – Mass-size relation when using more than one frequency
• More arctic in situ data sets required (ACME-V!)

Thank you for your attention!
Apply PAMTRA to ISDAC data

PAMTRA forward operator
- Simulates full Doppler spectrum
- T-Matrix for backscattering

Assumptions
- Elliptical particle shape with aspect ratio of 0.6

Radar obs.
- Equivalent radar reflectivity factor $Z_e$
- Mean Doppler velocity $W$
- Doppler spectrum width $\sigma$
- Skewness $Sk$
- Kurtosis $Ku$
- Slopes $Sl$ and $Sr$

Aircraft data
- Temperature $T$
- Pressure $p$
- Kinematic broadening $\sigma_k$
- Vertical wind $w$
- Particle size distribution $N(D)$
- Particle cross section area $A(D)$

Particle mass $m(D)$
$Z_e - W$ relation for $m(D)$

$W = e^{(Z_e)^f}$

Measured by MMCR!
$Z_e - W$ relation for $m(D)$

Using optimal estimation, choose $m(D)$ such that MMCR observations are matched.

Forward modelled ISDAC data
Exploit temperature dependence

\[ m(D) = a \ D^b \]
Validate simulation

Aircraft data
- Temperature $T$
- Pressure $p$
- Kinematic broadening $\sigma_k$
- Vertical wind $w$
- Particle size distribution $N(D)$
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MMCR Reference

Statistically

2016 ARM/ASR PI Meeting
Maximilian Maahn
May 2 – 6, 2016
Compare ISDAC and MMCR

High agreement, because retrieved m(D) is used

Also agreement of higher moments is high, problems with right slope: rare, large particles
Validate simulation

Aircraft data
- Temperature $T$
- Pressure $p$
- Kinematic broadening $\sigma_k$
- Vertical wind $w$
- Particle size distribution $N(D)$
- Particle cross section area $A(D)$

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- Particle mass $m(D)$
- $Z_e - W$ relation

Retrieval

Replace measurements with parameterizations

MMCR Reference

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2016 ARM/ASR PI Meeting

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Compare ISDAC and MMCR

Using the correct parameterizations, agreement is still high.
Set up idealized retrieval

- **Aircraft data**
  - Temperature $T$
  - Pressure $p$
  - Kinematic broadening $\sigma_k$
  - Vertical wind $w$
  - Particle size distribution $N(D)$
  - Particle cross section area $A(D)$

  **PAMTRA forward operator**
  - Simulates full Doppler spectrum
  - T-Matrix for backscattering

- **Radar obs.**
  - Equivalent radar reflectivity factor $Z_e$
  - Mean Doppler velocity $W$
  - Doppler spectrum width $\sigma$
  - Skewness $Sk$
  - Kurtosis $Ku$
  - Slopes $Sl$ and $Sr$

- **Assumptions**
  - Elliptical particle shape with aspect ratio of 0.6

- **Retrieval**
  - Particle mass $m(D)$
  - $Z_e - W$ relation

- **Optimal Estimation**
  - 8 state variables
  - 7 observables

**Set up idealized retrieval**

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How to simulate a Doppler spectrum (1)

Required variables:
N(D) (3p)

Particle Size
Distribution
N(D), e.g.
Gamma
distribution
How to simulate a Doppler spectrum (2)

Required variables:

- \( N(D) \) (3p)
- \( m(D) \) (2p)
- AR (1p)
- T (1p)

Single scattering properties, e.g. T-Matrix, depend on temperature \( T \), particle mass \( m(D) \) and aspect ratio \( AR \)
How to simulate a Doppler spectrum (3)

Required variables:
- \( N(D) \) (3p)
- \( m(D) \) (2p)
- AR (1p)
- T (1p)
- A(D) (2p)
- p (1p)

Fall velocity relation \( v(D) \), depends on cross section area \( A(D) \), \( m(D) \), pressure \( p \) and \( T \)

\[ \eta(D) \rightarrow \eta(v) \]
How to simulate a Doppler spectrum (4)

Required variables:
- N(D) (3p)
- m(D) (2p)
- AR (1p)
- T (1p)
- A(D) (2p)
- p (1p)
- U (1p)
- kin (2p)

depends (among others) on horizontal wind U and Eddy dissipation rate $\varepsilon$

apply vertical air motion and kinematic broadening
How to simulate a Doppler spectrum (5)

Required variables:
- \( N(D) \) (3p)
- \( m(D) \) (2p)
- \( AR \) (1p)
- \( T \) (1p)
- \( A(D) \) (2p)
- \( p \) (1p)
- \( U \) (1p)
- \( \text{kin} \) (2p)

depends on radar specifications (receiver, integration time etc.)

apply radar noise
Required data

In total 13 required quantities!
Not all of them equally important

Required variables:
- N(D) (3p)
- m(D) (2p)
- AR (1p)
- T (1p)
- A(D) (2p)
- p (1p)
- U (1p)
- kin (2p)