Comparison of stochastic collection equation solution methods for use in forward simulation of cloud radar Doppler spectra

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Introduction

- Cloud radar Doppler spectra provide rich information for evaluating the fidelity of particle size distributions from cloud models.
- Bin microphysics schemes develop particle size distributions more organically than bulk microphysics schemes, but they face the difficulty of numerical diffusion leading to overly rapid large drop formation, particularly while solving the stochastic collection equation (SCE).
- An accurate method for solving the SCE is essential for bin microphysics schemes to accurately simulate Doppler spectra, which rely on the sixth moment of the size distribution.

Experimental setup

Methods for solving the SCE

- ✓ J94 (Jacobson et al. 1994)
- ✓ BR74 (Berry and Reinhardt 1974)
- ✓ B00 (Bott 2000)

Large-Eddy Simulation

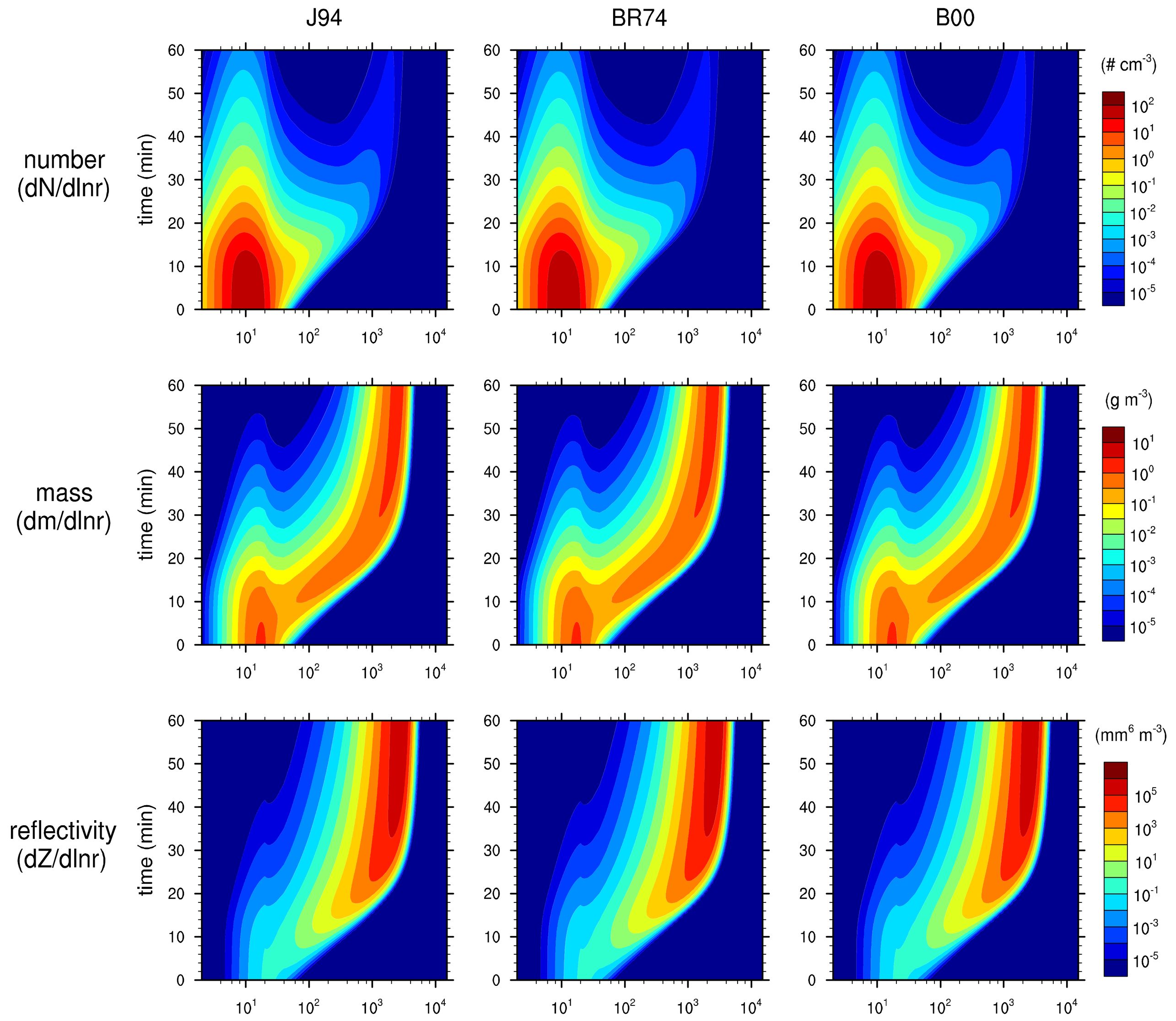
- ✓ previous modeling study: Rémillard et al. 2017
  - case: CAP-MBL campaign
  - numerical model: DHARMA
- ✓ methods for solving the SCE: J94 and B00
- ✓ drop mass bin: doubled at every two bins (number of bins: 70)
- ✓ time step: 3–5 sec (adaptive)
- ✓ initial aerosol number concentration: 65 cm^{-3}

Box model

- ✓ considering only collision-coalescence
- ✓ collection kernel: Hall (1980)
- ✓ initial DSD: a gamma distribution (LWC = 1 g m^{-3}, N_d = 100 cm^{-3})
- ✓ drop mass bin: doubled at every s bins (number of bins: 40×s)
- ✓ time step: 1–10 sec
- ✓ model integration time: 60 min

Box model: converged solution

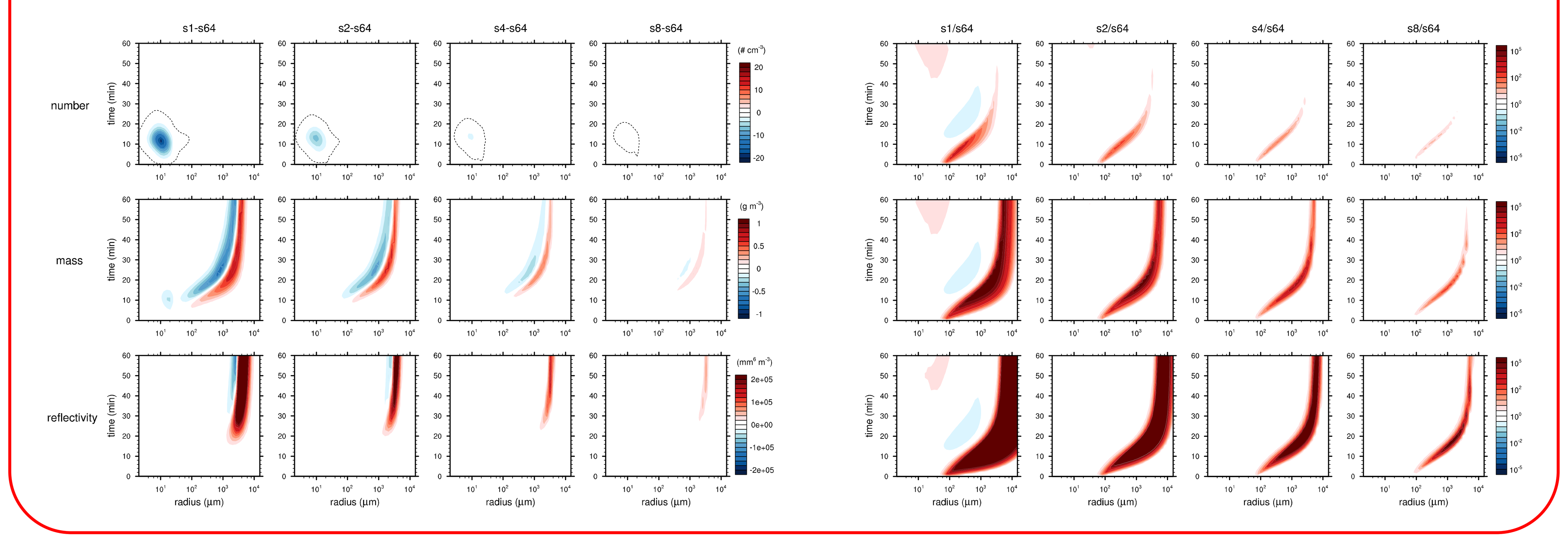
Time evolution of the moments of drop size distribution (s = 64, # of bins = 2560, Δt = 1 s)



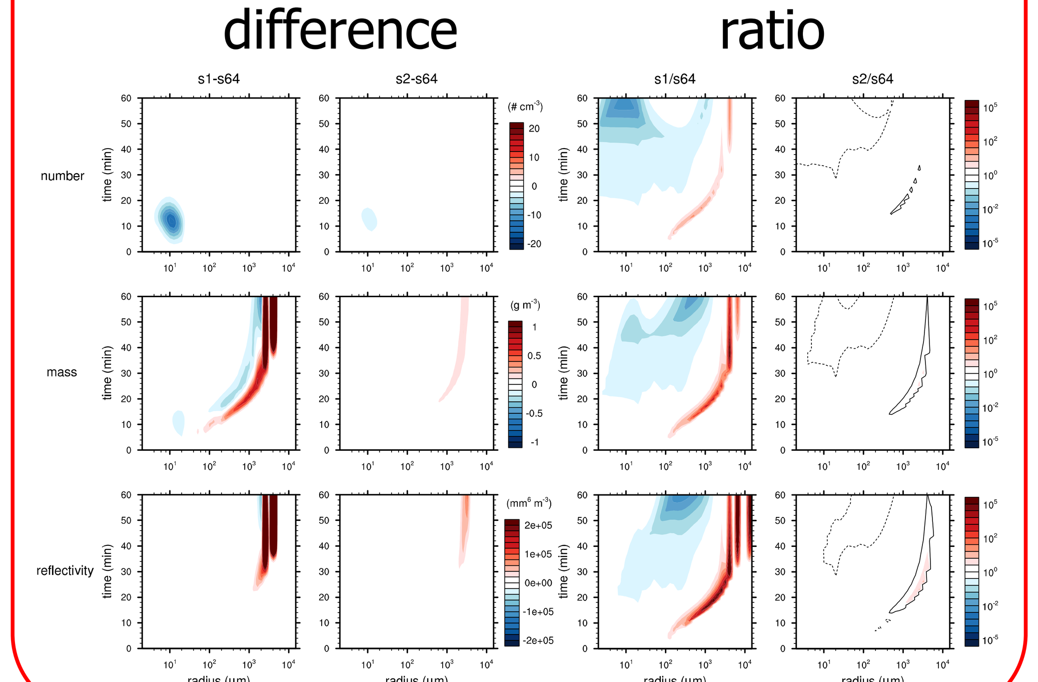
All the methods yield almost identical results at a sufficiently high resolution.

Convergence test: bin width

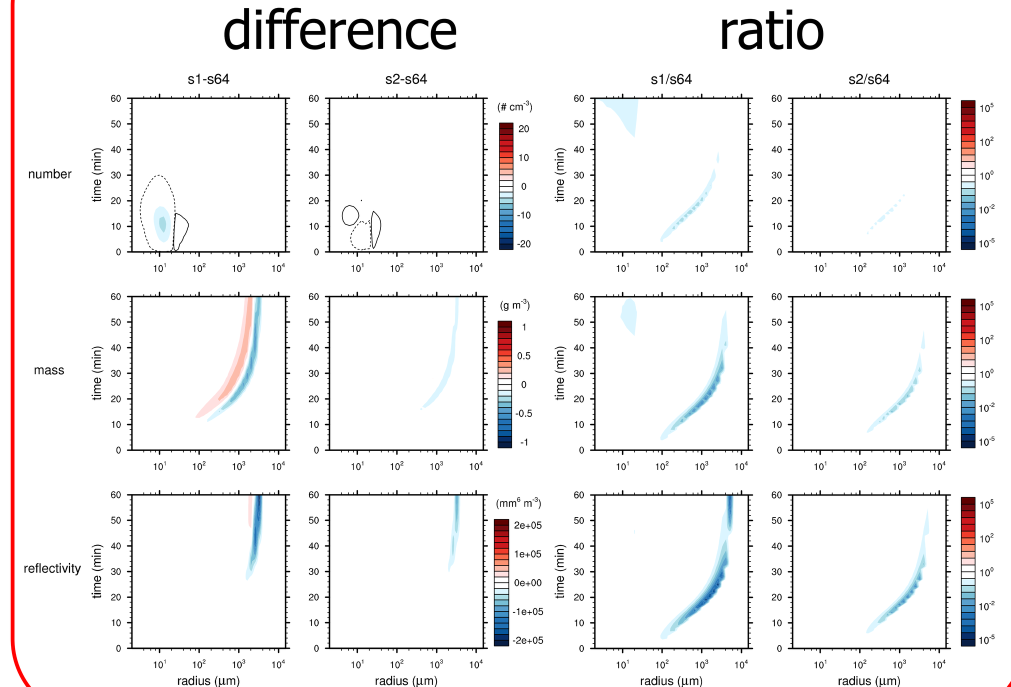
J94 (s = 1, 2, 4, 8)



BR74 (s = 1, 2)



B00 (s = 1, 2)



- The J94 method yields a dispersed solution even at a relatively high resolution.
- The BR74 and B00 methods yield the nearly converged solutions when $s \geq 2$.
- When s = 1, the B00 method is the most converged among the three methods.

number of bins: 40×s
time step: 1 sec
contour: ±0.1 cm^{-3} (difference)
1.5/0.667 (ratio)

Conclusions

- While all the methods for solving the SCE yields the almost identical converged solution, **B00 shows the best performance** among the examined methods (yielding a near-converged solution at a low resolution and computationally efficient).
- Experiment results obtained using B00 are more consistent with observations than those of J94.

References

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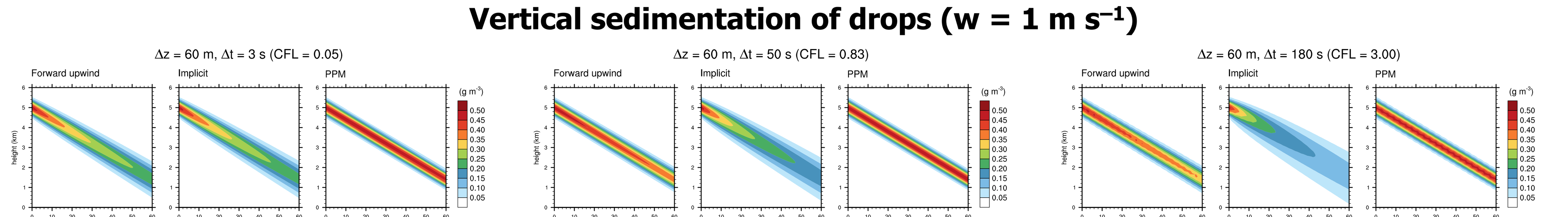
Sedimentation tests

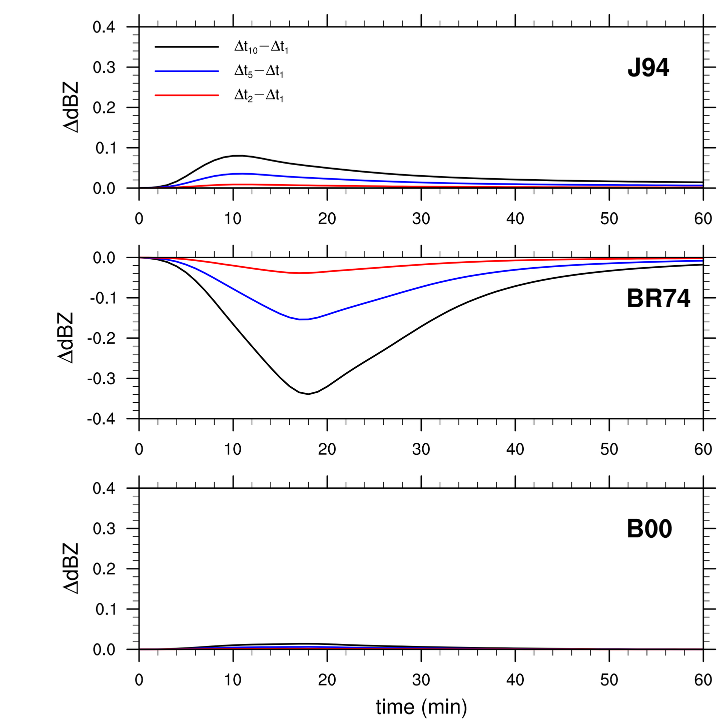
Vertical sedimentation of drops (w = 1 m s^{-1})



The PPM (Piecewise Parabolic Method, Colella and Woodward 1984) with sub-timestepping works best under all considered time steps, unlike a forward explicit upwind method or an implicit method.

Time step

Times series of radar reflectivity factor (difference from that with Δt = 1 s)

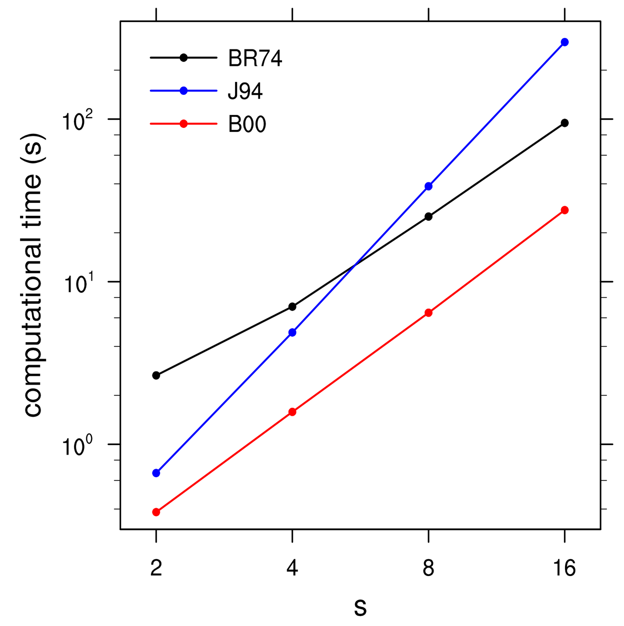


(number of bins: 80, unit of time step: sec)

While we find good performance of all methods, in contrast to Wang et al. (2007), B00 demonstrates substantially better convergence behavior than BR74.

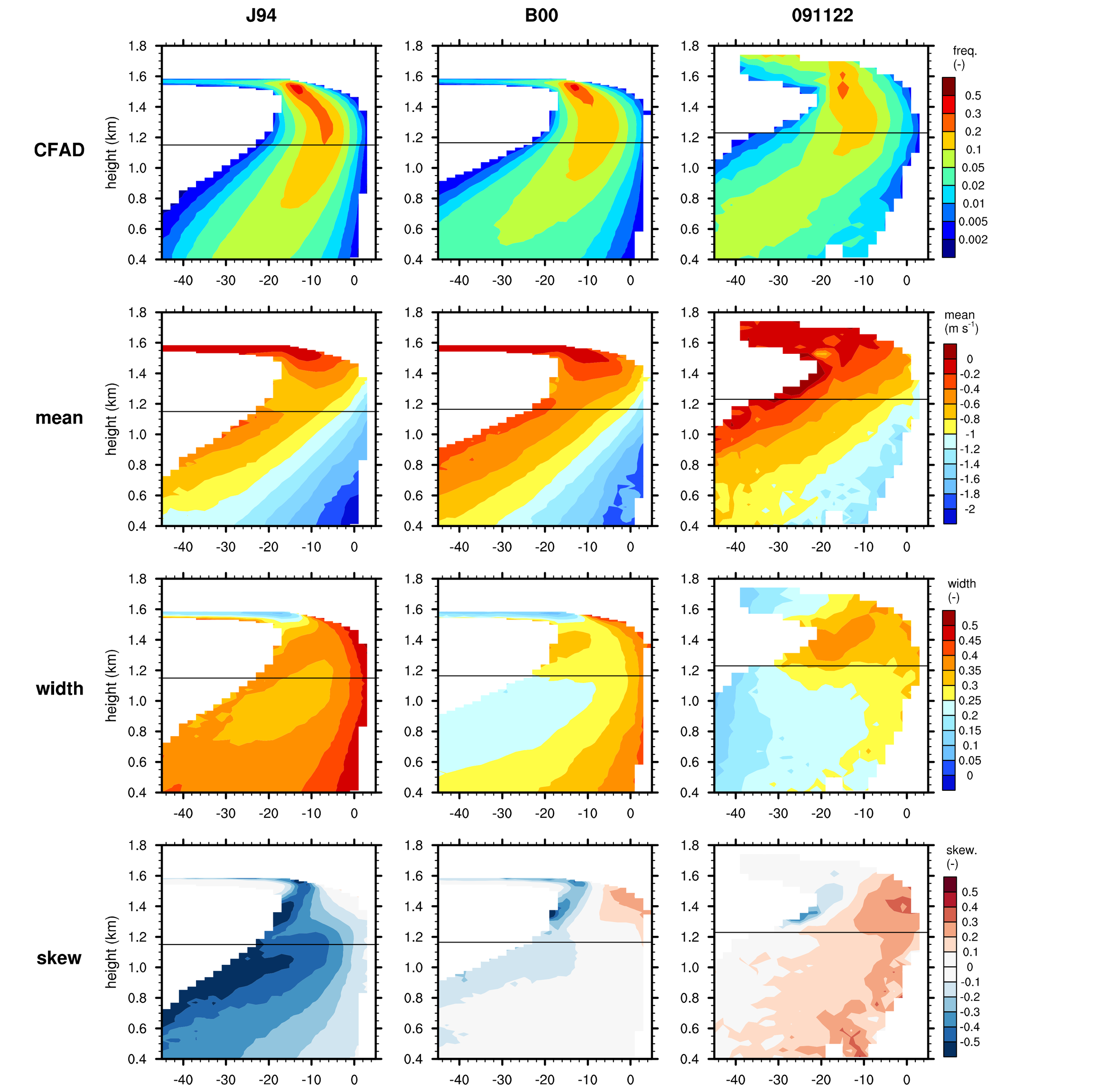
Efficiency

Time required for calculation as a function of number of bins



- BR74 and B00: ~$O(n^2)$, J94: ~$O(n^3)$.
- B00 is the most efficient among the three methods.

LES results vs. observation



(horizontal lines: mean cloud base)

B00 reduces mean Doppler velocity and Doppler spectra dispersion, and corrects too negatively skewed spectra shown in the J94 case, in better agreement with radar observations. However, the spectra simulated using B00 still show large mean velocity, wide dispersion, and negatively biased skewness.