On Spectral Invariance of Single Scattering Albedo at Weakly Absorbing Wavelengths

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The ratio \( \omega_\lambda(r)/\omega_\lambda(r_0) \) of two single scattering albedo spectra is a linear function of \( \omega_\lambda(r) \), whose slope and intercept sum to unity \((r = \text{eff. radius})\).

Water droplets

The Proof

Assumptions

- Scattering efficiency factor does not vary with either wavelength or droplet size;
- \( k_r < 0.1 \) where \( k_r \) is the bulk absorption coefficient (4\( \pi \) times ratio of imaginary refractive index to wavelength)

2 steps of the proof

(1) Co-albedo is proportional to droplet size (Twomey and Bohren, 1980): \( 1 - \omega_\lambda(r) = (2/3) k_r r \)

(2) Ignoring second order term \((|k_r|^2 r^2)\):

\[
\frac{\omega_\lambda(r)}{\omega_\lambda(r_0)} = \frac{p_\lambda + \sigma(1-p) + \sigma_a(r_0)}{\sigma_\lambda + \sigma_a(r)}
\]

If \( p = 1 - r_0/r \), the second term in the above equality can be written as

\[
\frac{\sigma(1-p) + \sigma_a(r_0) r}{\sigma_\lambda + \sigma_a(r)} = \frac{(1-p) + \sigma_a(r)(1-p)}{1 + \sigma_a(r)} = 1 - p
\]

Applications

- This relationship represents any single scattering albedo spectrum \( \omega_\lambda(r) \) via one known spectrum \( \omega_\lambda(r_0) \).
- Interpretation of spectrally invariant relationships in zenith radiances observed near cloud edges by the ARM SWS.

Summary

- For water droplets and ice crystals at weakly absorbing wavelengths, the ratio \( \omega_\lambda(r)/\omega_\lambda(r_0) \) of two single scattering albedo spectra is a linear function of \( \omega_\lambda(r) \), where \( r = \text{effective radius} \).
- The slope and intercept of the linear function are wavelength independent, sum to unity and depend only on \( r \).

Why?

Slope \( p \) as a function of effective radius \( r \) for different ice crystal habits and the approximation \( p_{\text{approx}} = 1 - r_0/r \) where \( r_0 = 5 \) \( \mu \)m since the regression is poorer when all 4 wavelengths are used.

More wavelengths

(left) \( \lambda \) between 0.2 \( \mu \)m and 4 \( \mu \)m with 10 nm spectral resolution; \( r = 20 \mu \)m, \( r_0 = 10 \mu \)m. Grey open dots correspond to wavelengths between 2.5 and 4 \( \mu \)m while black filled dots to wavelengths between 0.2 and 2.5 \( \mu \)m. The linear fit is for the black dots only. Red dots correspond to the 4 wavelengths: 0.86, 1.65, 2.13 and 3.75 \( \mu \)m. (right) \( \omega_\lambda(r) \) for \( r = 20 \mu \)m (grey dots) and the bulk absorption coefficient (black dots) as a function of wavelength (from Fig. 2.25 of Bohren and Clothiaux, 2006).