On Spectral Invariance of Single Scattering Albedo at Weakly Absorbing Wavelengths

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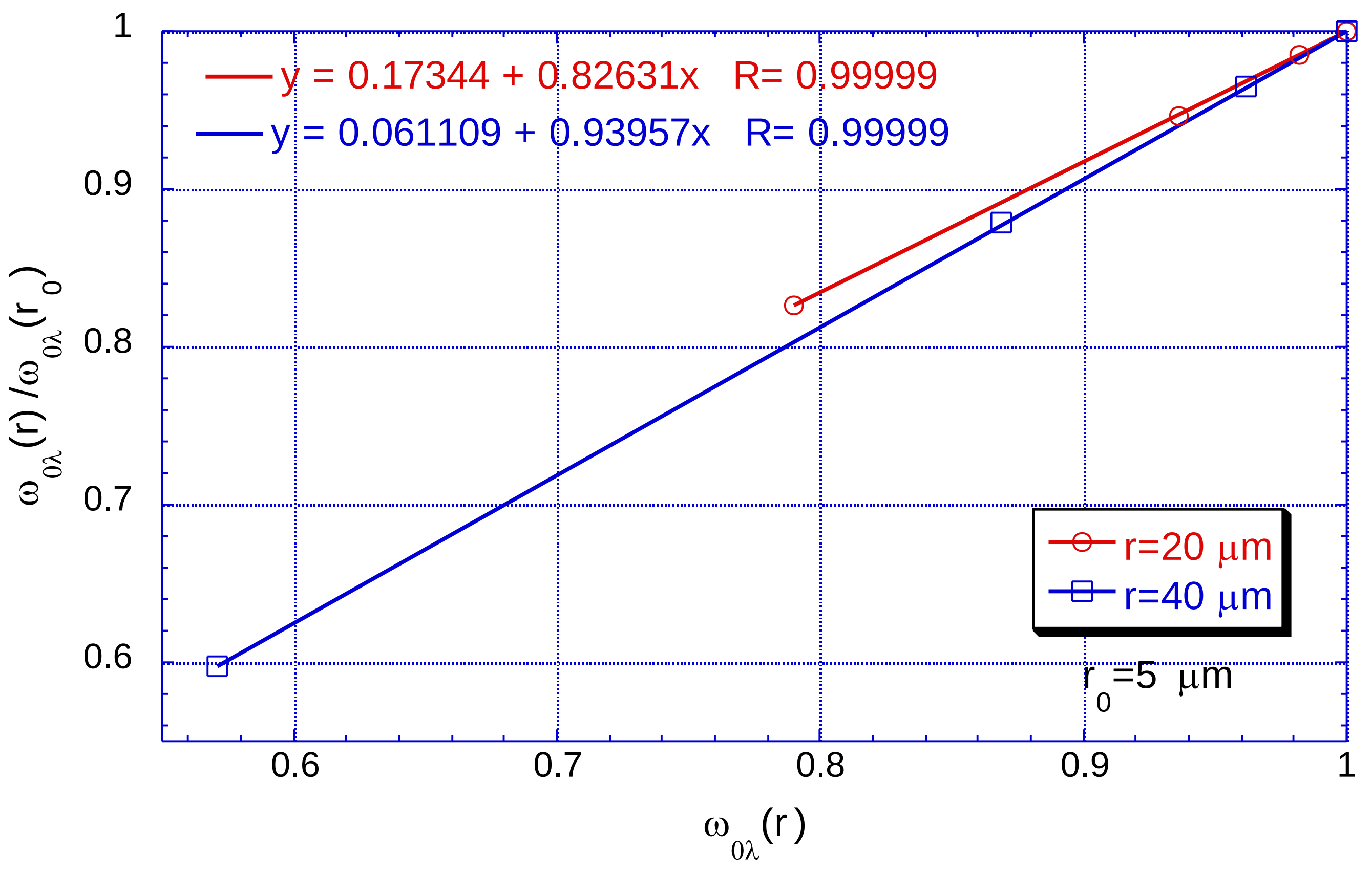
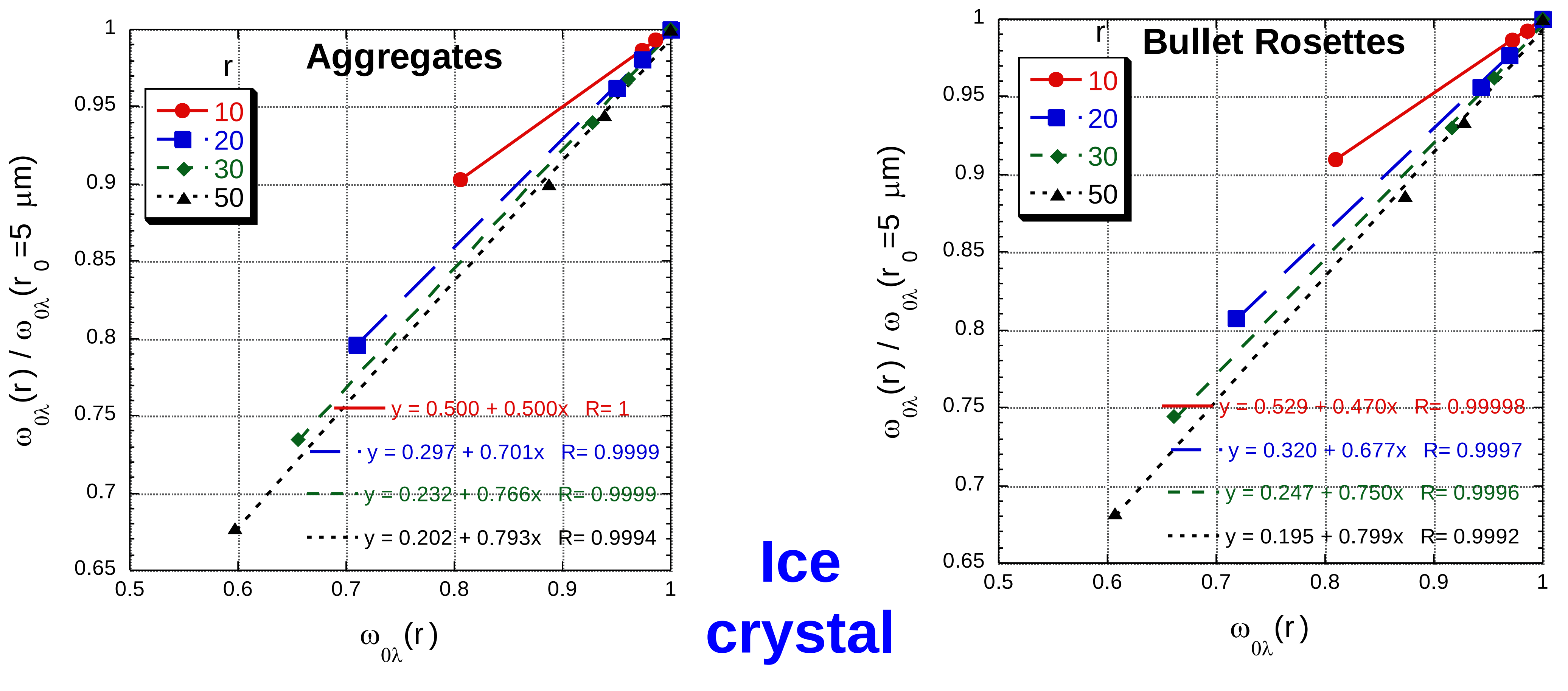
The ratio $\omega_{0\lambda}(r)/\omega_{0\lambda}(r_0)$ of two single scattering albedo spectra is a linear function of $\omega_{0\lambda}(r)$, whose slope and intercept sum to unity (r = eff. radius)



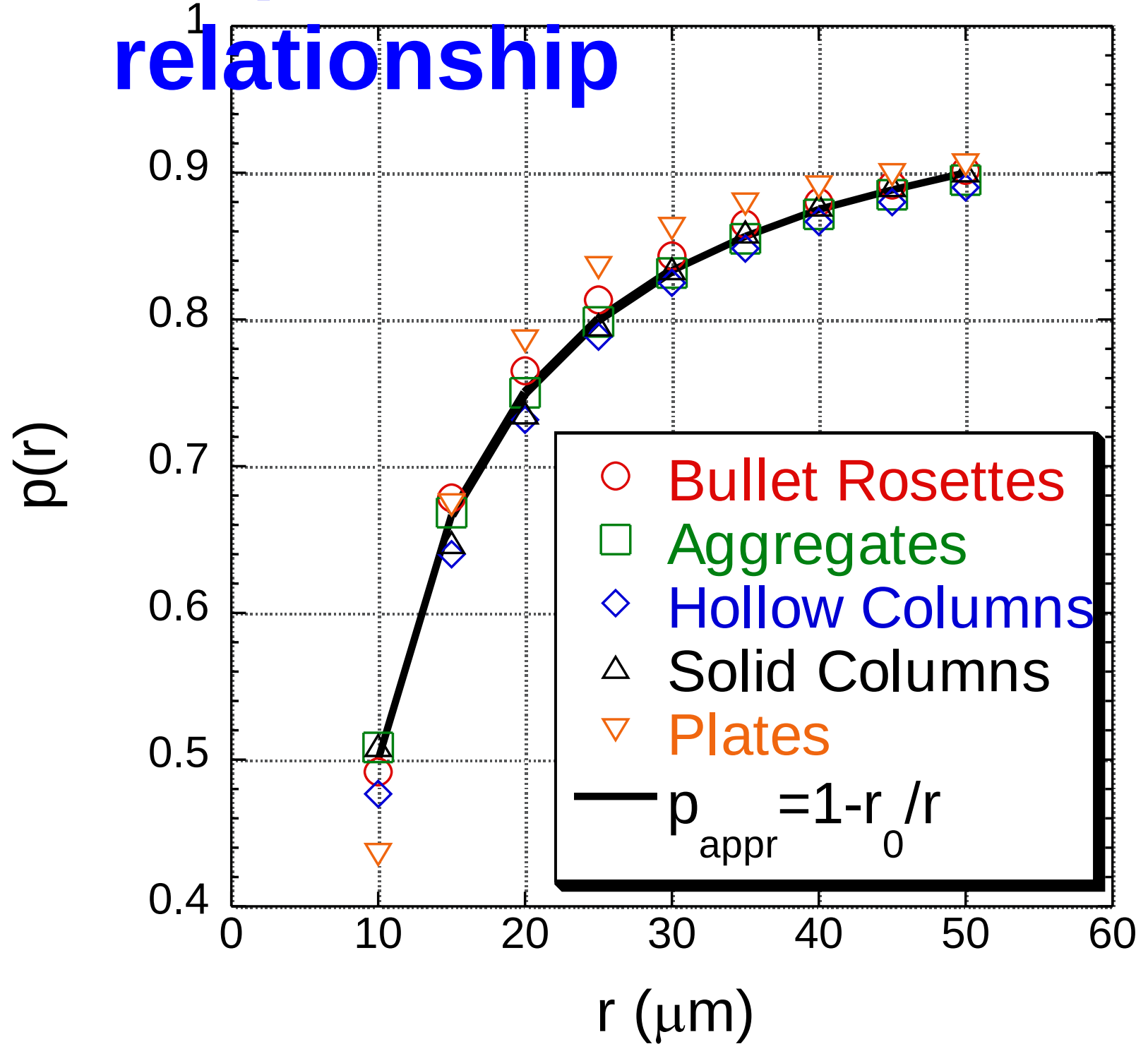
Water droplets

Single scattering albedo ratio $\omega_{0\lambda}(r)/\omega_{0\lambda}(r_0)$ vs. $\omega_{0\lambda}(r)$ for 4 wavelengths λ=0.86, 1.65, 2.13 and 3.75. Droplet effective radii r=20 & 40 μm; r_0=5 μm. Droplet size distributions are gamma with effective variance v=0.1 μm. Single scattering albedos calculated from Mie theory.



Single scattering albedo ratio $\omega_{0\lambda}(r)/\omega_{0\lambda}(r_0)$ vs. $\omega_{0\lambda}(r)$ for 4 wavelengths λ=0.86, 1.65, 2.13 and 3.75. Single scattering albedos from Ping Yang database (e.g., Yang et al., 2003). Particle effective radii r=10, 20, 30 and 50 μm; r_0=5 μm.

Slope of the linear relationship



Slope p as a function of effective radius r for different ice crystal habits and the approximation p_{appr}=1-r_0/r where r_0=5 μm. p is calculated using only three wavelengths (0.86, 1.65, and 2.13 μm) since the regression is poorer when all 4 wavelengths are used.

$$\frac{\omega_{0\lambda}(r)}{\omega_{0\lambda}(r_0)} = p\omega_{0\lambda}(r) + (1-p)$$

$$p = 1 - r_0/r$$

$$\omega_{0\lambda}(r) = \omega_{0\lambda}(r_0)\frac{1-p}{1-p\omega_{0\lambda}(r_0)}$$

Why ?

Summary

- For water droplets and ice crystals at weakly absorbing wavelengths, the ratio $\omega_{0\lambda}(r)/\omega_{0\lambda}(r_0)$ of two single scattering albedo spectra is a linear function of $\omega_{0\lambda}(r)$, where r = effective radius.
- The slope and intercept of the linear function are wavelength independent, sum to unity and depend only on r.

Applications

- This relationship represents <u>any</u> single scattering albedo spectrum $\omega_{0\lambda}(r)$ via one known spectrum $\omega_{0\lambda}(r_0)$.
- Interpretation of spectrally invariant relationships in zenith radiances observed near cloud edges by the ARM SWS.

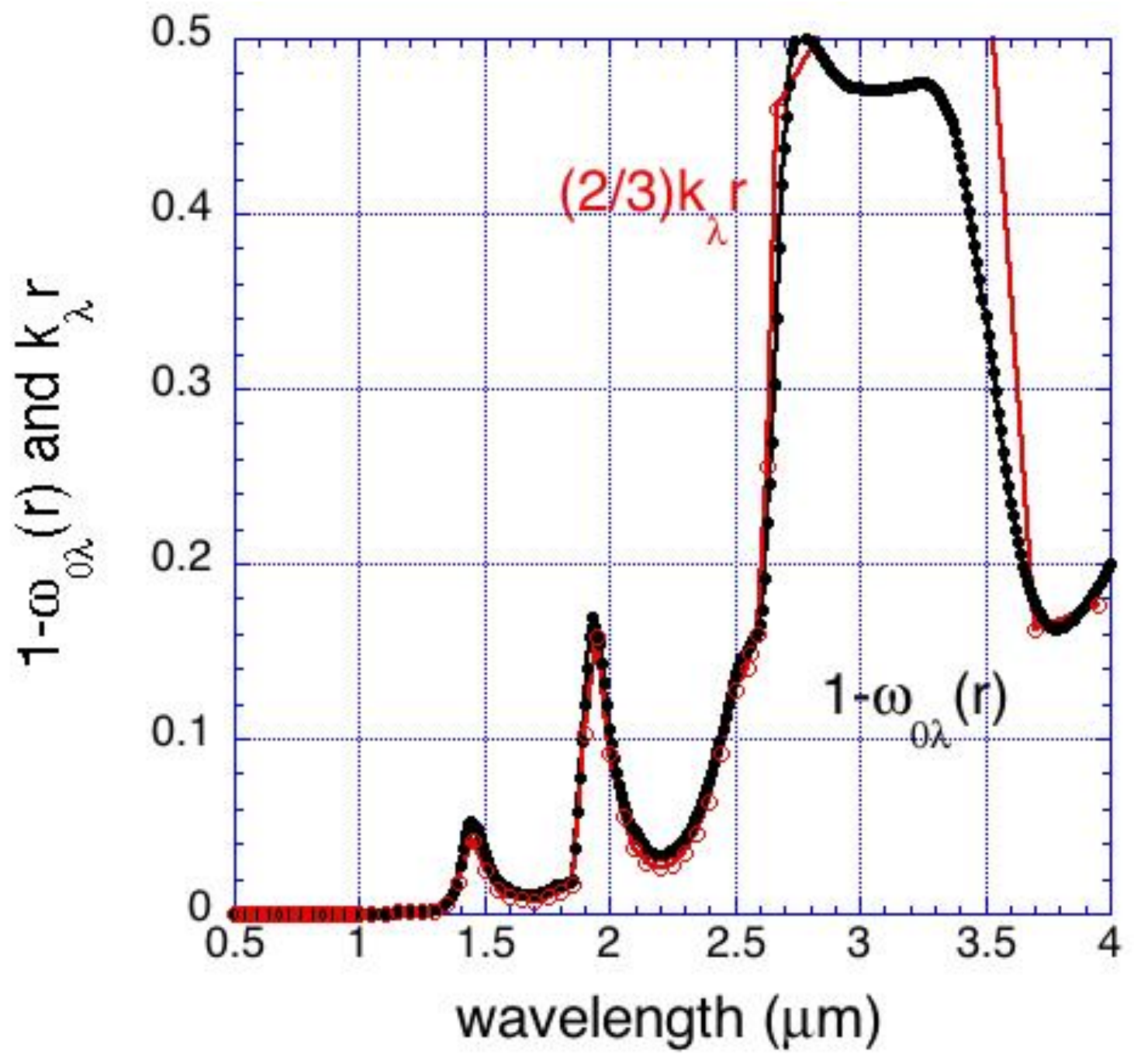
The Proof

Assumptions

- Scattering efficiency factor does not vary with either wavelength or droplet size;
- $k_\lambda r$ < 0.1 where k_λ is the bulk absorption coefficient (4π times ratio of imaginary refractive index to wavelength)

Two steps of the proof

(1) Co-albedo is proportional to droplet size (Twomey and Bohren, 1980): $1-\omega_{0\lambda}(r) \approx (2/3)k_\lambda r$



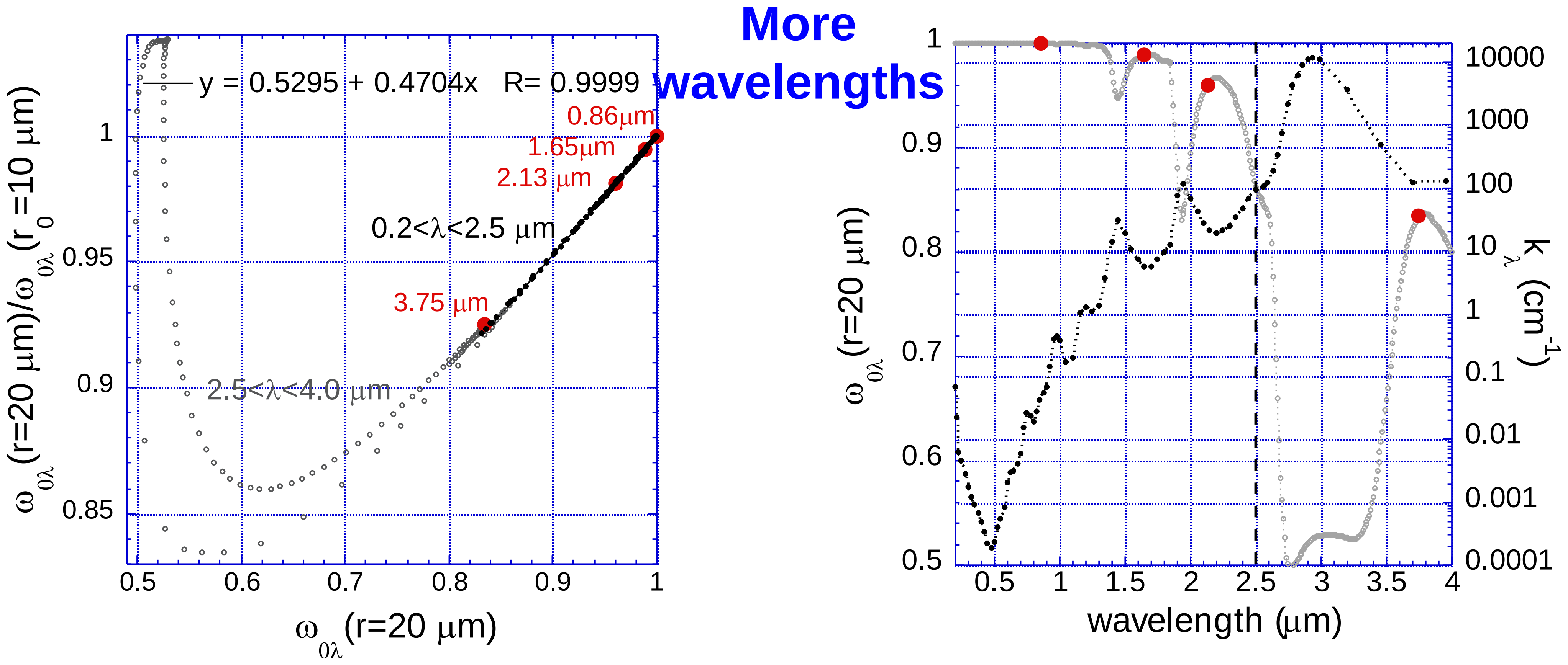
(2) Ignoring second order term $[(k_\lambda r)^2]$:

$$\frac{\omega_{0\lambda}(r)}{\omega_{0\lambda}(r_0)} = \frac{\sigma_s p + \sigma_s(1-p) + \sigma_{a\lambda}(r_0)}{\sigma_s + \sigma_{a\lambda}(r)} = p\omega_{0\lambda}(r) + \frac{\sigma_s(1-p) + \sigma_{a\lambda}(r_0)}{\sigma_s + \sigma_{a\lambda}(r)}$$

If p =1-r_0/r, the second term in the above equality can be written as

$$\frac{\sigma_s(1-p) + \sigma_s c k_\lambda r_0}{\sigma_s + \sigma_s c k_\lambda r} = \frac{(1-p) + c k_\lambda r(1-p)}{1 + c k_\lambda r} = 1-p$$

More wavelengths



(left) λ between 0.2 μm and 4 μm with 10 nm spectral resolution; r=20 μm, r_0=10 μm. Grey open dots correspond to wavelengths between 2.5 and 4 μm while black filled dots to wavelengths between 0.2 and 2.5 μm. The linear fit is for the black dots only. Red dots correspond to the 4 wavelengths: 0.86, 1.65, 2.13 and 3.75 μm . **(right)** $\omega_{0\lambda}(r)$ for r=20 μm (grey dots) and the bulk absorption coefficient (black dots) as a function of wavelength (from Fig. 2.25 of Bohren and Clothiaux, 2006).