Extending 94 GHz Radar Retrievals of Ice Water Content beyond the Rayleigh Regime

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Abstract

The main findings of this study are (1) analytical formulations based on Rayleigh scattering can be extended to size parameter x ≈ 0.3 (and possibly more) for ice crystal aggregates using the Rayleigh-Gans Approximation (RGA) developed by Westbrook (2006, 2008) and (2) our results agree within a factor of 1.6 with those of Szyrmer et al. (2012, JGR). Moreover, for both ice spheres and snowflakes, significant departure from Rayleigh scattering occurs around x = 0.16. Regarding finding (1), the ice particle size distribution (PSD) median mass dimension ($D_{m,ice}$) and effective diameter (D_e) may reach ~ 500 µm and 200-260 µm, respectively, before exceeding the range of applicability of the RGA formulation. We have formulated the Westbrook RGA in terms of ice particle mass-dimension relationships, which allowed us to compare our results with those of Szyrmer et al. (2012) who used a forward model based on Mie theory and variable density spheres. The relative agreement between these two techniques having very different approaches appears encouraging. The approach used here is analytical and relatively simple to apply, using ice particle mass-dimension relationships from field observations.

Model Description: Rayleigh Gans Approximation (RGA)

From Rayleigh scattering theory, the ice water content (IWC) can be expressed in terms of the ice particle size distribution (PSD) slope λ, the ice particle shape and the equivalent radar reflectivity factor Z_e as described in Mitchell et al. (2006). Adding the RGA form factor F that allows us to extend this relationship to higher size parameters x,

$$IWC = \frac{\alpha\,\Gamma(\beta+\nu+1)(g_w/g_i)\,Z_e\,\lambda^{\beta}}{\Gamma(2\beta+\nu+1)F}$$

where g_w/g_i is for unit conversions and ice particle mass is given as

$$m = \alpha D^{\beta}$$

and the PSD is a gamma function:

$$N(D) = N_o D^{\nu} \exp(-\lambda D) .$$

The form factor for a single snowflake is given by Westbrook as

$$f = (1 + 0.159 x^2) / (1 + (0.159 + 1/3) x^2 + 0.164 x^4)$$

where x=2kr, k is the wavenumber 2π/Λ, Λ = wavelength, and r = radius of gyration = 0.3 D where D = maximum ice particle dimension. In our case Λ = 3.2 mm. In this way the form factor F that represents the entire PSD can be calculated for any type of PSD by integrating f over the PSD and weighting f as described in Westbrook (2006):

$$F = \frac{\int f\, m^2\, N(D)\, dD}{\int m^2\, N(D)\, dD} .$$

Through this weighting F depends on ice particle shape (i.e. α and β) and the PSD shape. This approach may differ substantially in predicted Z_e from the bulk approach described in Westbrook (2006), where F implicitly assumes the PSD predicted from Westbrook's aggregation model.

The basis of the RGA is that the electromagnetic field within the particle can be approximated by the incident field, and each sub-volume of the particle produces the Rayleigh type scattering independently from other volume elements. The RGA is applicable when the following two conditions are satisfied:

$$|m-1| << 1 \;\; and \;\; 2x|m-1| << 1$$

where m is refractive index and x = size parameter.

The RGA calculations presented here have been adapted to conform with the retrieval framework and microphysical methodology of Szyrmer et al. (2012, JGR). The Westbrook form factor F is based on a fractal dimension (β) of 2 that was found for ice crystal aggregates. Thus our results are based on β = 2, which is similar to the β of many ice particle types. We have related the Z_e/IWC ratio to size parameter x, $2kr_{av}$ (r_{av} = average radius of gyration), effective diameter D_e, ice particle median mass dimension $D_{m,ice}$ and the mean mass-weighted diameter of the "melted" PSD of mass-equivalent spheres D_m in order to gain a more comprehensive physical understanding of this RGA treatment and its impact on the IWC-Z_e relationship. Relating Z_e / IWC to D_m allows us to compare our results with those of Szyrmer et al. (2012).

Dependence of Form Factor on Ice Particle Shape

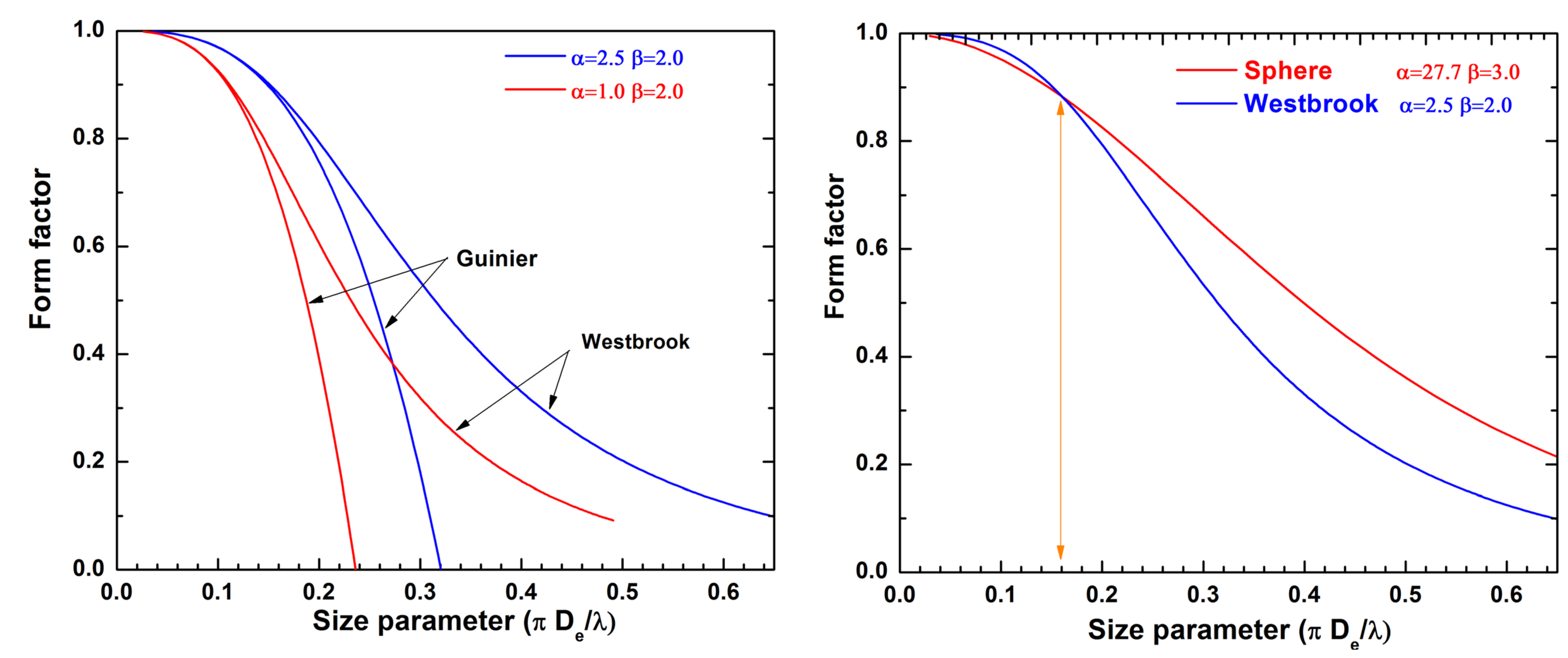

Fig.1. Form factor as a function of the size parameter x, where D_e is the effective diameter of ice and $F_{Guinier} \approx 1 - 1/3(2Kr_{av})^2$. The form factor F indicates the deviation from the Rayleigh scattering. Left panel shows the Guinier and Westbrook form factors for low and high density aggregates, where $D_e = \int D^3 N(D) dD / \int D^2 N(D) dD$ and the PSD is for melted mass-equivalent spheres. The Guinier form factor is only valid for size parameter x < 0.14. The right panel shows the form factor calculated for high-density aggregates and ice spheres. The vertical line shows where Rayleigh scattering begins to break down. Note how F progressively broadens as ice particle shape changes from low- to high-density aggregates, and then to ice spheres.

Z_e / IWC : Dependence on ice particle size and size parameter

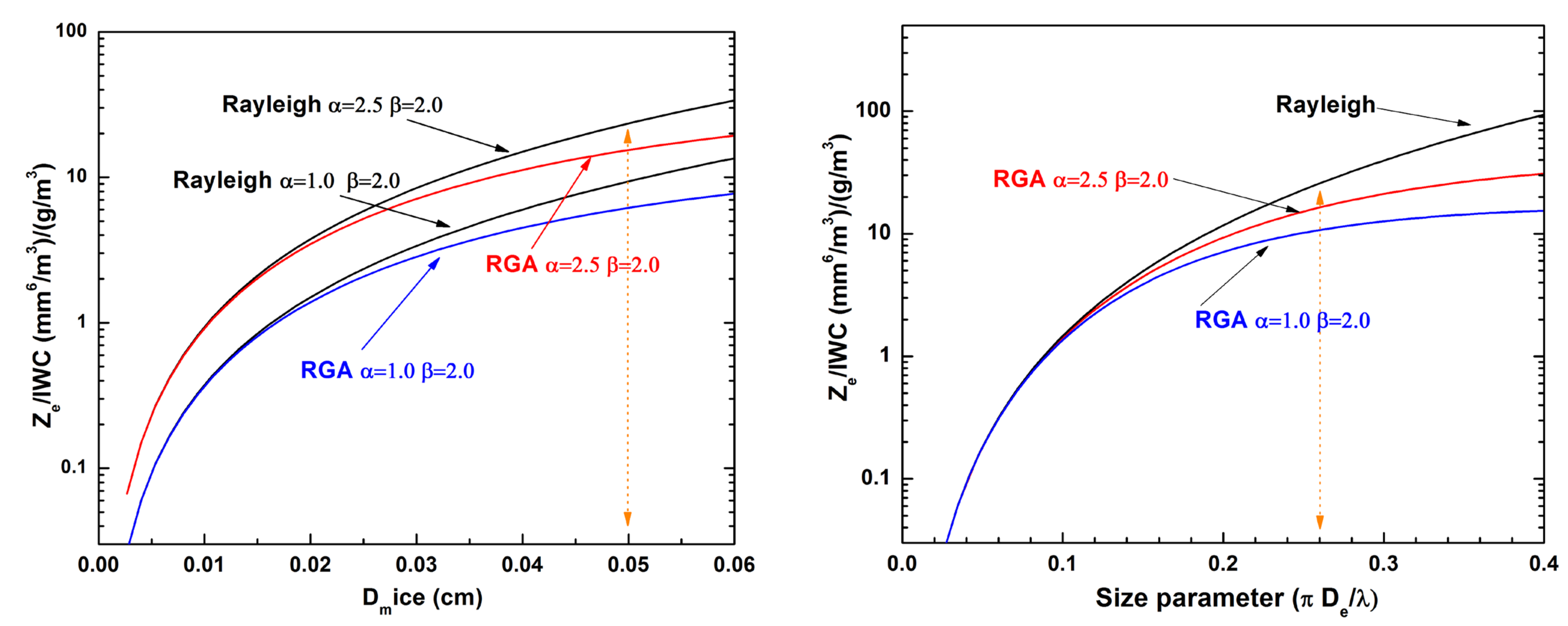

Fig. 3. Left panel: Dependence of Z_e/IWC on the ice PSD median mass dimension for low density (α = 1.0) and high density (α = 2.5) ice crystal aggregates using the RGA. Corresponding black curves are predicted from the same equation but without F and are based on Rayleigh scattering. Right panel: Dependence of Z_e/IWC on the size parameter for the RGA and Rayleigh scattering. The orange dotted line in both panels shows the RGA range of validity.

Z_e / IWC: Comparison between RGA and Rayleigh formulations

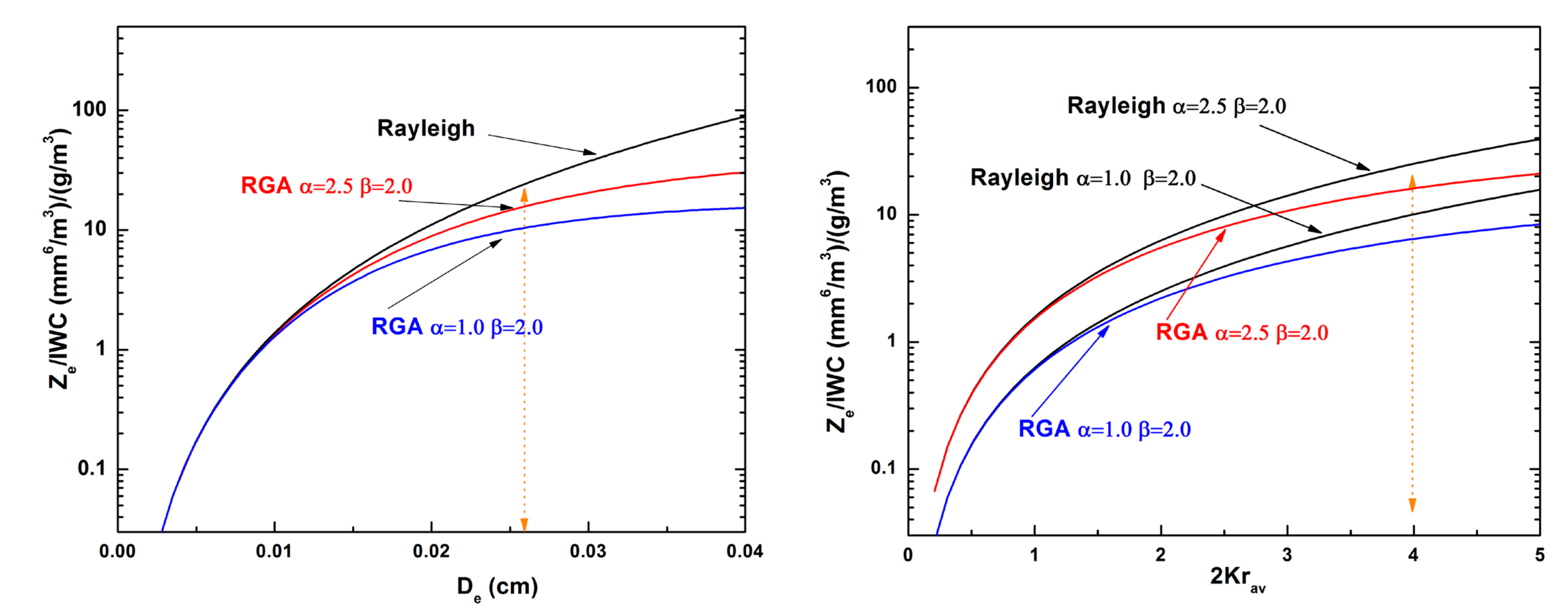

Fig. 2. Relating the ratio of effective radar reflectivity Z_e to ice water content, Z_e/IWC, to the PSD effective diameter and the quantity $2kr_{av}$ where k = wavenumber = 2π/Λ and r_{av} = average radius of gyration ($r_{av} \approx 0.3\ \bar{D}$ where $\bar{D}$ = mean aggregate length). The dotted orange line with arrows indicates the range of validity of the Westbrook form factor, with larger values not considered in the simulated aggregation data used by Westbrook. The black curves show Z_e/IWC calculated without form factor F (i.e. F = 1.0) included in the forward model (governing equation shown here).

In all figures, the m-D prefactor α' is normalized following the methodology of Szyrmer et al. (2012): $\alpha = \alpha'\, C_o\, (1/D^*)^{\beta}$ where $m = \alpha' D^{\beta}$, $C_o = 3 \times 10^{-5}$ g and D* = 0.12 cm. This was done to compare our results with those of Szyrmer et al. (2012).

Z_e / IWC: Comparison with Szyrmer et al. (2012)

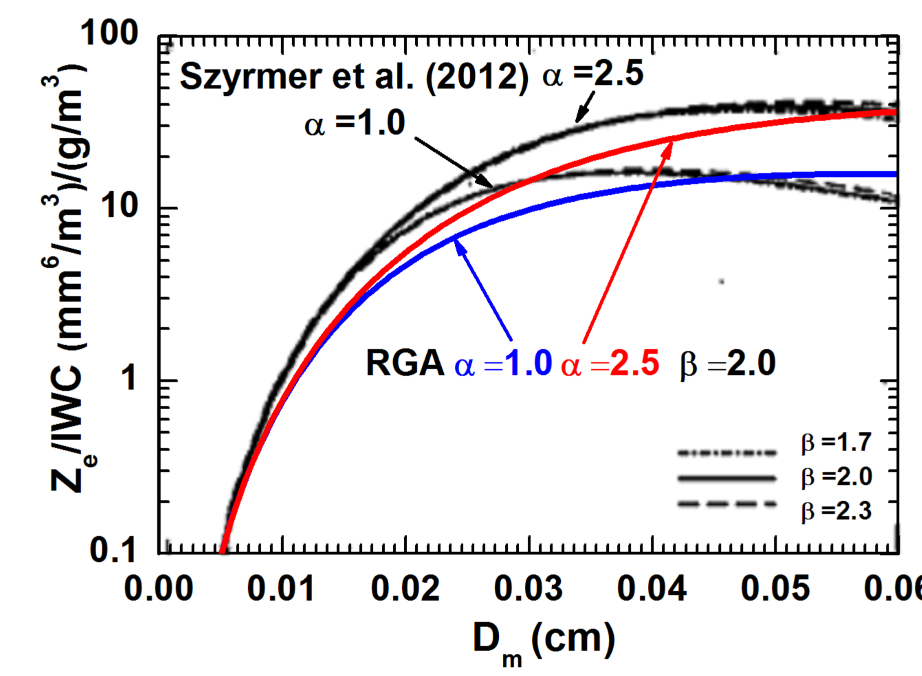

Fig. 4. Comparison of Z_e/IWC predicted from this RGA formulation with that predicted using the method of Szyrmer et al. (2012, JGR). This required relating Z_e/IWC to the PSD characteristic size D_m (mean mass weighted diameter of the equivalent liquid water sphere PSD; 4th PSD moment/3rd moment). The scattering forward model of Szyrmer et al. is based on Mie theory and approximates snowflakes as ice spheres having lower density in their outer shells. Presumably the variable ice densities depend on α and β, and both approaches use the same values of α and β. The Szyrmer et al. results are consistent with our results within a factor of 1.6. Considering the differences in forward models, this agreement appears relatively good. Neither the Szyrmer et al. results nor our results are sensitive to differences in PSD shape.

Conclusions

- Using the Westbrook form factor for single ice particles, our RGA formulation allows us to estimate Z_e/IWC out to x ≈ 0.3. This corresponds to D_e ≈ 260 µm and an ice PSD median mass dimension ≈ 500 µm. This should be adequate for most cirrus cloud conditions but not adequate for most frontal cloud conditions. However, it is possible that our RGA formulation is valid for significantly larger x and D_e and more research is needed to determine the limits of this formulation.
- Our predicted values of Z_e/IWC are within a factor of 1.6 to those of Szyrmer et al. (2012, JGR) for a given PSD characteristic size. While their method used a very different forward model for backscattering, both studies used the same mass-dimension power law assumptions, and for both studies Z_e/IWC was not sensitive to the PSD shape (i.e. ν) for a given characteristic size and ice particle shape. This suggests that when x is well beyond the Rayleigh scattering limit and when a characteristic ice particle size is accurately retrieved through another measurement, then one of the greatest challenges in radar retrievals of IWC may lie in characterizing the ice particle mass-dimension power law for the clouds being sampled.

References

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