The main findings of this study are (1) analytical formulations based on Rayleigh scattering can be extended to size parameter x = 0.3 (and possibly more) for ice crystal aggregates using the Rayleigh-Gans Approximation (RGA) developed by Westbrook (2006, 2008) and (2) our results agree within a factor of 1.6 with those of Szyrmer et al. (2012, JGR). Moreover, for both ice spheres and snowflakes, significant departure from Rayleigh scattering occurs around x = 0.16. Regarding finding (1), the ice particle size distribution (PSD) median mass dimension (Dm,ice) and effective diameter (D0) may reach ~ 500 μm and 200-260 μm, respectively, before exceeding the range of applicability of the RGA formulation. We have formulated the Westbrook RGA in terms of ice particle mass-dimension relationships, which allowed us to compare our results with those of Szyrmer et al. (2012) who used a forward model based on Mie theory and variable density spheres. The relative agreement between these two techniques having very different approaches appears encouraging. The approach used here is analytical and relatively simple to apply, using ice particle mass-dimension relationships from field observations.

Abstract

Model Description: Rayleigh Gans Approximation (RGA)

From Rayleigh scattering theory, the ice water content (IWC) can be expressed in terms of the ice particle size distribution (PSD) slope λ, the ice particle shape and the equivalent radar reflectivity factor F as described in Mitchell et al. (2006). Adding the RGA form factor F that allows us to extend this relationship to higher size parameters x,

\[ IWC = \frac{2}{3} \frac{\beta^3}{\lambda^4} \frac{m}{N(D) dD} \]

where \( m = \frac{4}{3} \pi D^3 \) is for unit conversions and ice particle mass is given as

\[ m = \alpha \beta \gamma \lambda \]

and the PSD is a gamma function:

\[ N(D) = \frac{\beta^\lambda}{\Gamma(\lambda)} \exp(\alpha D) \frac{1}{D} \]

The form factor for a single snowflake is given by Westbrook as

\[ f = \frac{1}{1 + 0.159 x^2} \]

where \( x = 2 k r, k \) is the wavenumber \( 2 \pi / \lambda \), \( \lambda \) = wavelength, and \( r = \) radius of gyration = 0.3 D where D = maximum ice particle dimension. In our case \( \lambda = 32 \) mm. In this way the form factor F that represents the entire PSD can be calculated for any type of PSD by integrating F over the PSD and weighting f as described in Westbrook (2006):

\[ F = \frac{1}{m^2} \int m N(D) dD \]

\[ \int m^2 N(D) dD \]

Through this weighting F depends on ice particle shape (i.e. \( \alpha \) and \( \beta \)) and the PSD shape. This approach may differ substantially in predicted ZE from the bulk approach described in Westbrook (2006), where F implicitly assumes the PSD predicted from Westbrook’s aggregation model.

The basis of the RGA is that the electromagnetic field within the particle can be approximated by the incident field, and each sub-volume of the particle produces the Rayleigh type scattering independently from other volume elements. The RGA is applicable when the following two conditions are satisfied:

\[ |m - 1| < 1 \quad \text{and} \quad 2 \pi k r - 1 < 1 \]

where m is refractive index and x is size parameter.

The RGA calculations presented here have been adapted to conform to the bulk approach described in Westbrook (2006), 2008, and (2) our results agree within a factor of 1.6 with those of Szyrmer et al. (2012, JGR). Moreover, for both ice spheres and snowflakes, significant departure from Rayleigh scattering occurs around x = 0.16. Regarding finding (1), the ice particle size distribution (PSD) median mass dimension (Dm,ice) and effective diameter (D0) may reach ~ 500 μm and 200-260 μm, respectively, before exceeding the range of applicability of the RGA formulation. We have formulated the Westbrook RGA in terms of ice particle mass-dimension relationships, which allowed us to compare our results with those of Szyrmer et al. (2012) who used a forward model based on Mie theory and variable density spheres. The relative agreement between these two techniques having very different approaches appears encouraging. The approach used here is analytical and relatively simple to apply, using ice particle mass-dimension relationships from field observations.

Conclusions

- Using the Westbrook form factor for single ice particles, our RGA formulation allows us to estimate ZE/IWC out to x = 0.3. This corresponds to D0 = 260 μm and an ice PSD median mass dimension = 500 μm. This should be adequate for most cirrus cloud conditions but not adequate for most frontal cloud conditions. However, it is possible that our RGA formulation is valid for significantly larger x and D0 and more research is needed to determine the limits of this formulation.

- Our predicted values of ZE/IWC are within a factor of 1.6 to those of Szyrmer et al. (2012, JGR) for a given PSD characteristic size. While their method used a very different forward model for backscattering, both studies used the same mass-dimension power law assumptions, and for both studies ZE/IWC was not sensitive to the PSD shape (i.e. x) for a given characteristic size and ice particle shape. This suggests that when x is well beyond the Rayleigh scattering limit and when a characteristic ice particle size is accurately retrieved through another measurement, then one of the greatest challenges in radar retrievals of IWC may lie in characterizing the ice particle mass-dimension power law for the clouds being sampled.

References