Failure of Taylor's hypothesis in the atmospheric surface layer and its correction for eddy-covariance measurements

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1. Background

- Taylor's hypothesis plays a fundamental role in transforming commonly available temporal data into typically-inaccessible spatial spectrum [Kaimal et al, 1972].
- Turbulent eddies convect at same velocity irrespective of eddy size [Taylor, 1938].
- Turbulent convection velocity may be related to eddy size [Krogstad et al., 1998].
- Turbulent fluxes are underestimated in surface energy balance [Foken, 2008].

2. Objective

2.1 To determine the relationship between turbulent convection velocity and eddy size

High resolution spatial and temporal data are needed.

Previous studies were limited by spatial data length or spatial resolution.

2.2 To improve the eddy-covariance measurements by correcting the application of Taylor's hypothesis in retrieving wavenumber spectrum by frequency spectrum

3. Method

3.1 Experiment setup

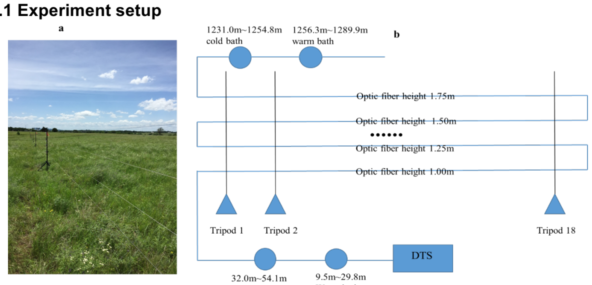

Figure 1. **a** Picture of the installation at the Oklahoma State University Range Research Station site. Four parallel 233-meter long optic fibers were maintained by tripods to measure air temperature in the direction of 50 degrees North of East . **b** Schematic of the DTS [Selker et al., 2006] transect and of the fiber optic setup. The four water baths were used to calibrate the optic fiber temperature.

DTS data

The temperature resolution was 0.16 ℃ and 0.22 ℃ before and after transect respectively, which was calculated from the calibration baths using the standard deviation of bath temperature. Spatial resolution was 0.56 m and temporal resolution was 3 s.

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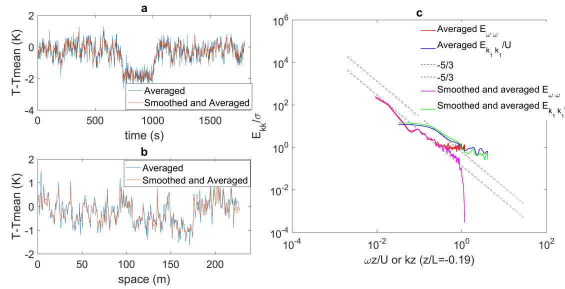

Figure 2. **a** "averaged" and "smoothed and averaged" temporal temperature data for a fixed point in the transect. **b** temperature along the transect for a fixed time. The y axis is temperature minus the mean. **c** Temperature spectra in the time and space domain. ω is frequency in the time domain, k is wavenumber in the spatial domain, k_1 is streamwise wavenumber, z is optic fiber height above ground, L is Obukhov length, U is mean wind velocity, $E_{\omega\omega}$ is power spectrum in frequency, E_{kk} is power spectrum in wavenumber, σ is temperature variance of temporal or spatial data.

3.2 Calculating convection velocity

$$U(k_1) = \frac{\Delta\varphi}{k_1 \times \Delta t}$$

k_1 is streamwise wavenumber, $U(k_1)$ is wavenumber dependent convection velocity, $\Delta\varphi$ is the phase difference of Fourier transform of velocity between time t and $t + \Delta t$. [Buxton et al., 2013]

4. Results

Convection velocity calculated by DTS data

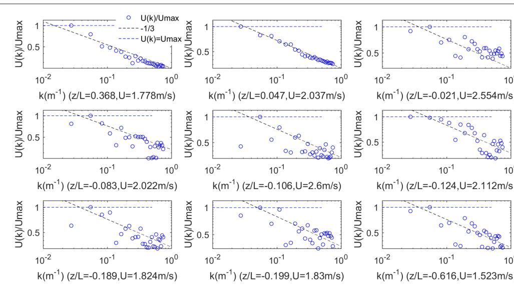

Figure 3. Ratios of different wavenumber components convection velocities and maximum convection velocity under different stability conditions. z is optic fiber height above ground, L is obukhov length, k is one-dimensional streamwise wavenumber, $U(k)$ is convection velocities of different wavenumber components, and U_{max} is maximum convection velocity, U is mean wind velocity. The maximum convection velocity rather than mean velocity is the denominator.

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Convection velocity calculated by LES

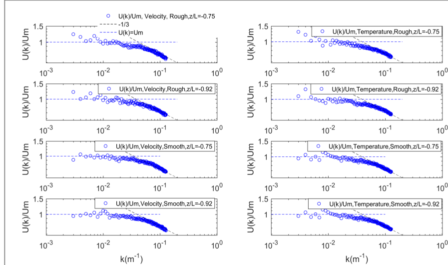

Figure 4. Ratios of different wavenumber components convection velocities and mean streamwise velocity of LES data. z is the vertical height of the LES data, L is Obukhov length, k is one-dimensional streamwise wavenumber, $U(k)$ is convection velocities of different wavenumber components, and U_m is mean streamwise velocity.

Scaling in the inertial subrange

$u(l) = (\epsilon l)^{\frac{1}{3}}$, $u(l)$ is the characteristic velocity scales for an eddy size l in the inertial subrange, ϵ is dissipation rate. [Kolmogorov, 1941][Pope, 2000]

Idealized random sweeping [*Kraichnan*, 1964] $\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{v}\nabla \boldsymbol{u} = 0$

Eddies in the large limit of the inertial subrange [*Wilczek and Narita*, 2012] $\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{U} + \boldsymbol{v})\nabla \boldsymbol{u} = 0$

Eddies in the small limit of inertial subrange $\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{v}\nabla \boldsymbol{u} = D\nabla^2 \boldsymbol{u}$, $\left|\frac{U_d}{U}\right| \propto k_1^{-1/3}$.

where $\boldsymbol{u}$ is small-scale velocity field (For eddies in the inertial subrange), t is time, $\boldsymbol{U}$ is constant mean velocity of large-scale motion, $\boldsymbol{v}$ is random sweeping velocity constant in time and space with a Gaussian ensemble distribution, D is turbulent diffusion coefficient related to eddy sizes in the inertial subrange.

Wavenumber-dependent convection velocity and wavenumber spectrum

Taylor's hypothesis

$$k_1 z = \frac{\omega z}{\overline{U}}$$

Corrected form

$$k_1 z = \frac{\omega z}{\overline{U}} \frac{\overline{U}}{U(k_1)} = \frac{\omega z}{U(k_1)}$$

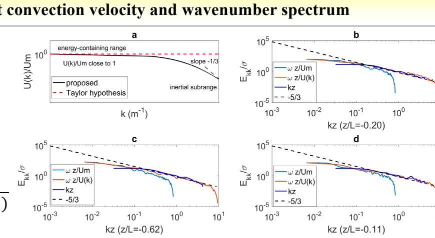

Figure 5. a Schematic relation between convection velocity and wavenumber indicated by Taylor's hypothesis and this paper. **b, c & d** Normalized frequency spectrum($\omega z/U_m$), corrected normalized frequency spectrum ($\omega z/U(k)$) and measured normalized one-dimensional wavenumber spectrum (kz). k is one-dimensional streamwise wavenumber in the spatial domain, $U(k)$ is convection velocity of different wavenumber component, U_m is mean streamwise wind velocity,ω is angular frequency, z is optic fiber height above ground, L is Obukhov length, E_{kk} is power spectrum in wavenumber, σ is temperature variance of spatial data or temporal data.

5. Conclusion

1. Convection velocity exhibits a $k_1^{-1/3}$ dependence in the inertial subrange.
2. Random sweeping causes decorrelation (frequency broadening) at high wavenumber, which does not violate mean convection velocity.
3. It is the "frozen" hypothesis that is not correct in [Taylor, 1938] because eddies in the small limit of inertial subrange are decaying fast due to turbulent diffusion as they are advected.
4. Taylor's hypothesis underestimates 10%-30% of turbulent energy in the inertial subrange.
5. A correction for eddy-covariance measurements is proposed.

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