Here, we explore analytic DSDs, those produced by a state-of-the-art bin model, and DSDs from ARM disdrometer observations.

**Analytic DSDs**
- Use the gamma DSD: \( N(D) = N_0 D^p \exp(-\frac{\epsilon D}{\Lambda}) \)
- Compute self-consistent \((M_k, M_j)\) pairs for \( k, j = 0, 20 \). From each \((M_k, M_j)\), two DSD parameters \((N_0, \Lambda)\) are obtained for a wide range of \( \epsilon \) similar to what has been observed.
- Compute \( Z_{\text{H}}, Z_{\text{DP}}, \) and \( K_{\text{DP}} \) from these DSDs using the T-matrix method (Fig. 1).

**ARM Disdrometer Data**
- **PARSIVEL-2** and 2D video disdrometer data from ARM sites around the world are used.

**Combined Data: Results**

- 20.7 million “realistic” DSDs; moments and dual-pole variables computed for each (Fig. 3).

**Bin Model Simulation Data**
- 1D bin microphysical model of Prat and Barros (2007, JAMC) used. Simulations run for 60 minutes (output \( \Delta t = 1 \) min) in a 3-km-tall domain (\( \Delta z = 10 \) m). Normalized gamma DSDs initialized at domain top with the following parameter ranges:
  - \( \Delta z = 0.2 \) mm to 4 mm
  - \( N_0 = 100 \) to 80000 mm\(^{-1}\) m\(^{-3}\)
  - \( \epsilon = 1 \) to 10
  - \( \Lambda = 0.01 \) mm\(^2\) hr\(^{-1}\) < \( R < 500 \) mm\(^2\) hr\(^{-1}\), resulting in 10742 simulations.
  - DSDs are taken at every output time and height, resulting approximately 199 million DSDs.

**Introduction**
- We are developing a novel warm-rain microphysics scheme (BOSS, P. Poster 119).
- BOSS uses Bayesian inference for robust parameter uncertainty estimation, which facilitates constraint by observations.
- Dual-polarization radar observations will provide a probabilistic constraint on scheme structure and microphysical sensitivities to environmental conditions.
- BOSS can use any combination of prognostic drop size distribution (DSD) moments. Unlike most schemes, however, it does not specify a DSD functional form.

This necessitates development of a moment-based polarimetric radar forward operator.

The \( k \)-th DSD moment (\( M_k \))

\[
M_k = \int_{D_{min}}^{D_{max}} N(D)D^k dD
\]

\( D_{min}, D_{max} \) minimum, maximum drop sizes
\( N(D) D^k \) number density of drops with diameters \( D \) to \( D + dD \).

Choice of prognostic moments will be partly based on the resultant uncertainty in our forward operator.

A given value of \( M_k \) can arise from an infinite number of DSDs. Our goal is to assess variability in the subset of realistic DSDs.

Here, we explore analytic DSDs, those produced by a state-of-the-art bin model, and DSDs from ARM disdrometer observations.

Define a combined variability parameter:

\[
T = \delta M + \delta \epsilon M_0 + \delta \epsilon M_1
\]

\( \delta \epsilon \) expected observation uncertainty
\( \epsilon \) standard deviation of drop variables for a given set of moments \((M_k, M_j)\) arising from DSD variability

Thus, the look-up table will include not only the mean values of \( Z_{\text{H}}, Z_{\text{DP}}, \) and \( K_{\text{DP}} \) in each bin, but also the de-trended standard deviation of \( Z_{\text{H}}, Z_{\text{DP}}, \) and \( K_{\text{DP}} \) within a bin, as well as the distribution skews, and covariances between the polarimetric radar variables.

For the 2-moment BOSS, we desire the pair of predicted moments \((M_k, M_j)\) that minimizes uncertainty in the forward operator (i.e., for which pair of moments do the dual-polarization radar variables provide the most information?)

Compute the distribution-weighted standard deviation for each radar variable \( X \):

\[
\xi = \sum_{i=1}^{N} \sigma_i [M_k(i), M_j(i)] \times P[M_k(i), M_j(i)]
\]

where \( P \) is the joint pdf of \( M_k \) and \( M_j \) which are discretized into M and N bins, respectively. Results for all \((k, j)\) pairs considered are shown in Fig. 4.

Formulate the forward operator for a number of different moment pairs (for 2-moment BOSS) or triads (for 3-moment BOSS). This will be in the form of a look-up table. Fig. 5 shows an example of what this looks like for \((M_0, M_1)\).

**Figures**
- Fig. 1: Vertical standard deviations of \( Z_{\text{H}}, Z_{\text{DP}}, \) and \( K_{\text{DP}} \)羽 winter from ARM data as a function of \( \epsilon \). Note the different color scales used in each panel. (a) shows the standard deviation of \( Z_{\text{H}} \) corresponding to enrolled points in top left panel. The blue-dashed line is for moment pair \((k, j)\), otherwise the gray-dashed line is for moment pair \((12, 3)\).

**Development of a Polarimetric Radar Forward Operator for the Bayesian Observationally- Constrained Statistical-physical Scheme (BOSS)**

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Funding for this research comes from the U.S. Department of Energy Atmospheric System Research (ASR) Program, award DE-SC0016579.