Developing high-resolution constrained variational analysis of vertical velocity and advective tendencies within the range of ARM scanning radars at the SGP

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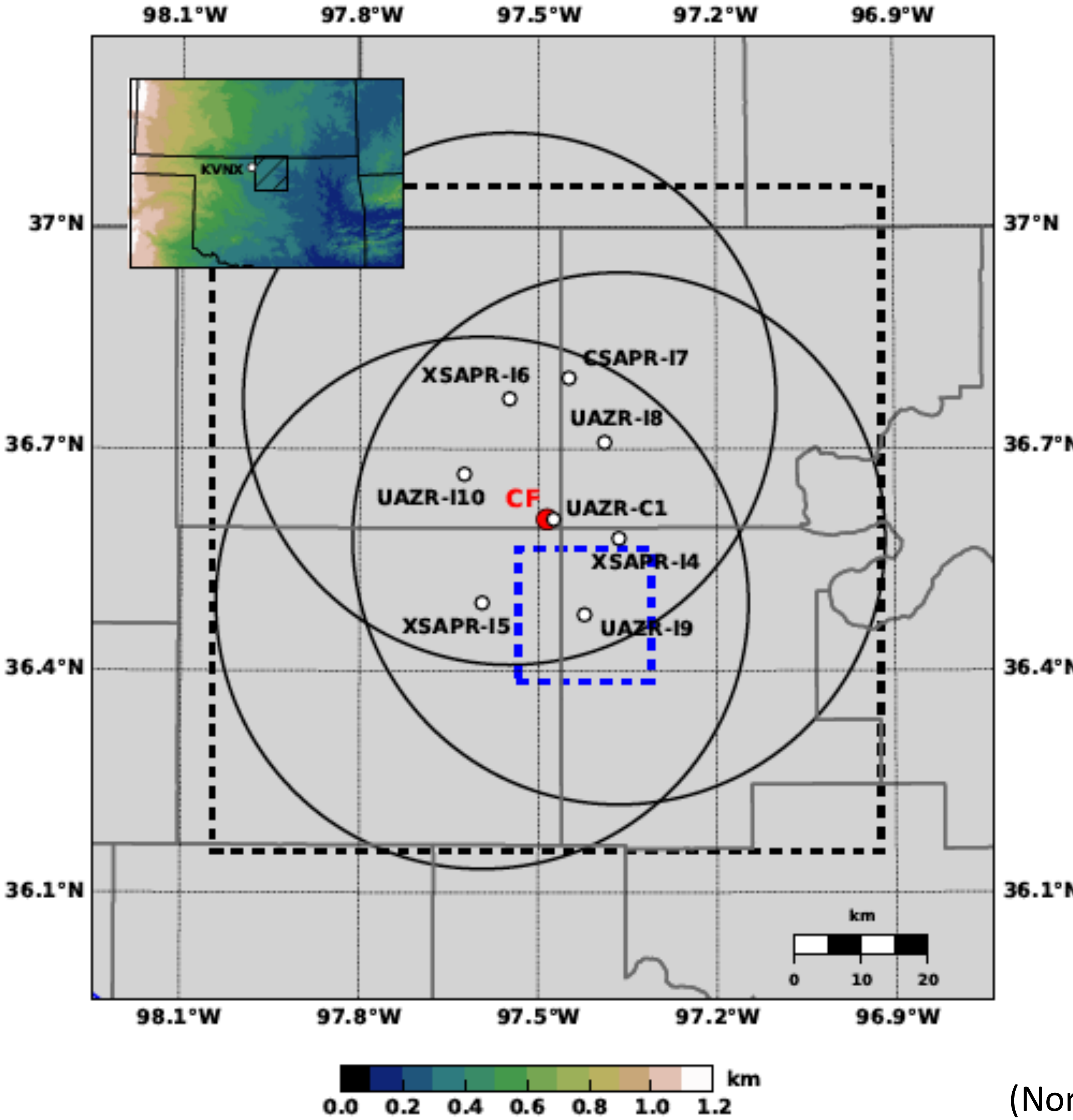
Acknowledgements: Shaocheng Xie and Shuaiqi Tang


98.1°W
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97.5°W
97.2°W
96.9°W
37°N
36.7°N
36.4°N
36.1°N
KVNX
CSAPR-I7
XSAPR-I6
UAZR-I8
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XSAPR-I5
UAZR-I9
km
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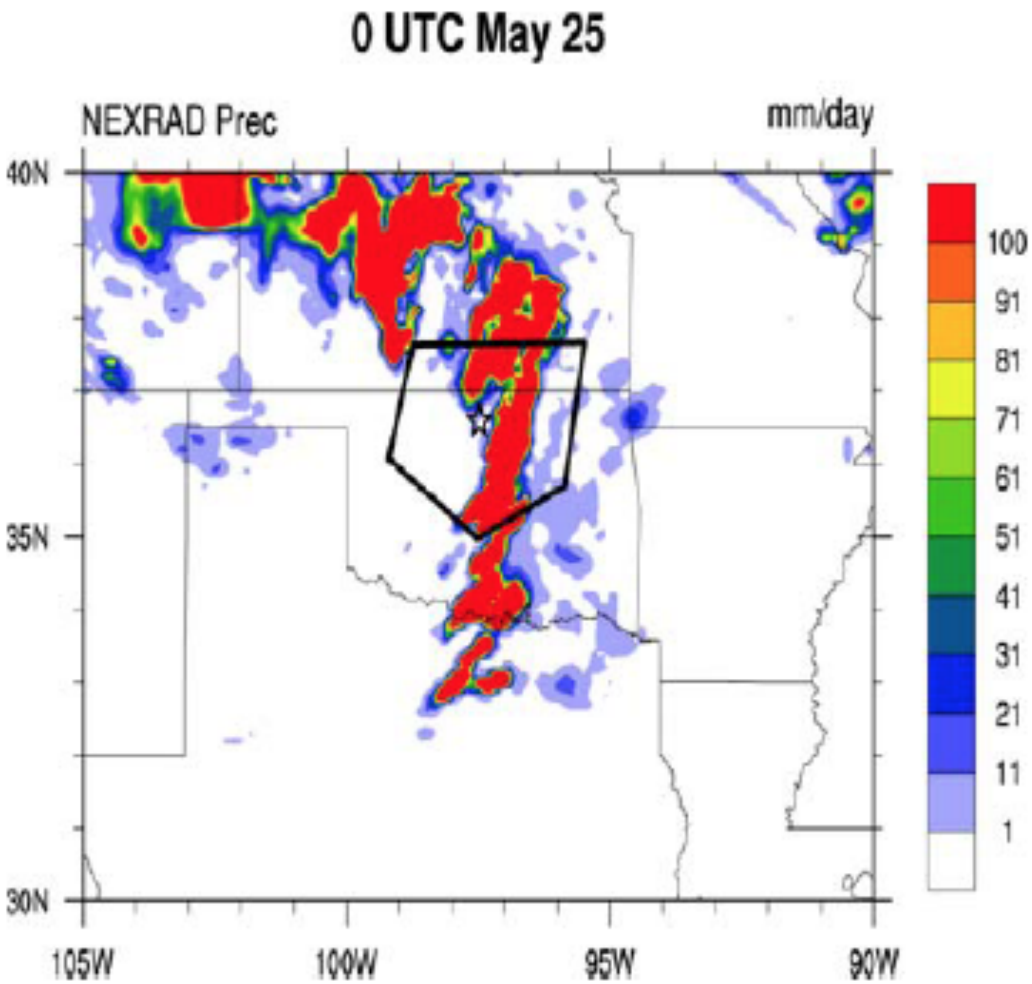
1.2

(North 2015)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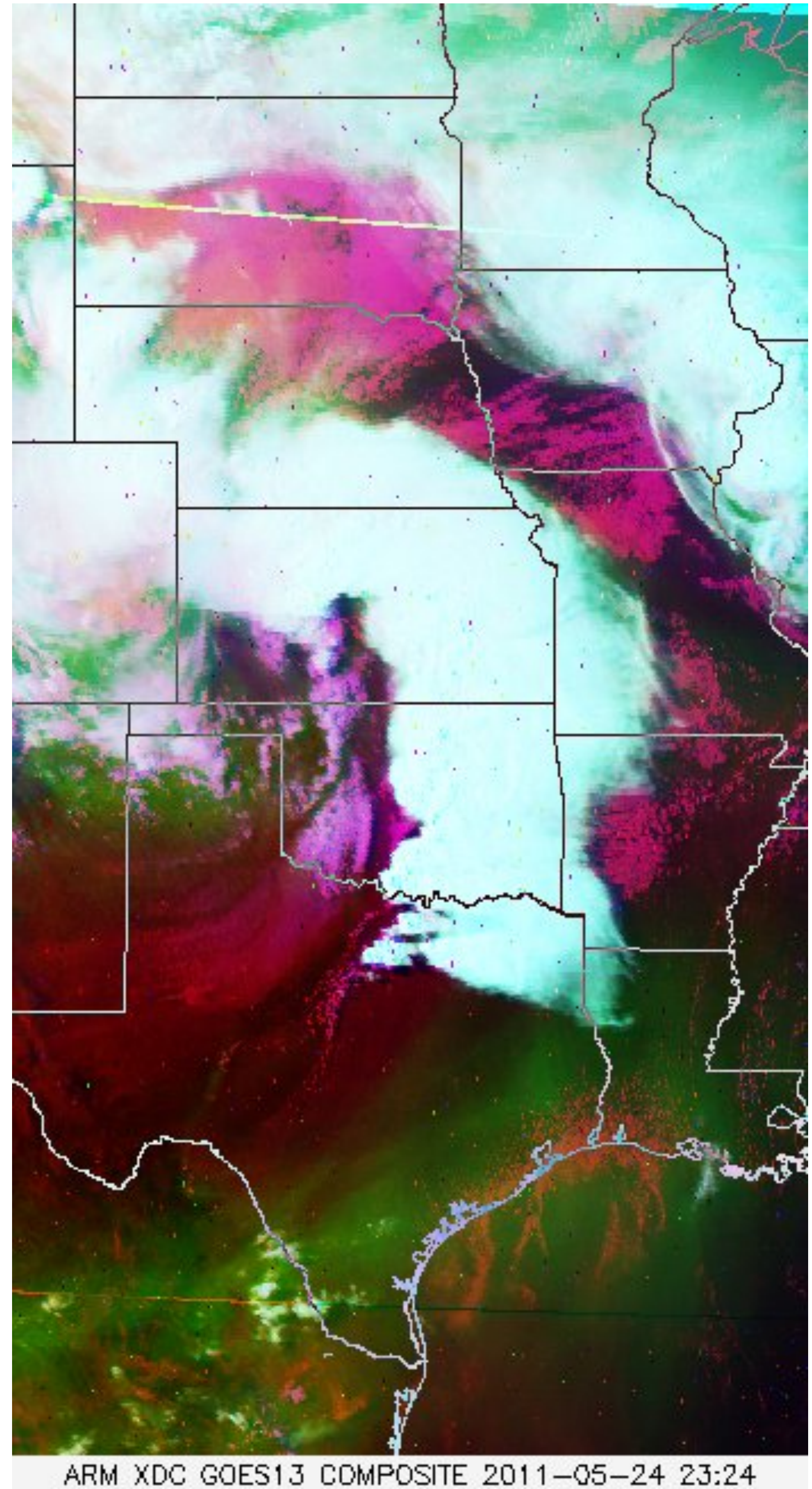

ARM XDC GOES13 COMPOSITE 2011-05-24 23:24

Objective:

- To derive atmospheric dynamical and thermodynamic fields that are consistent/compatible with observed physical variables of precipitation, radiation, and turbulent heat fluxes at 3-4 km scale resolution.
- Vertical velocity, horizontal winds, advective tendencies

Three methods:

1. Radar retrievals (e.g., North and Kollias 2015)
2. Operational data assimilation (GSI) or multiscale-data assimilation (Li et al. 2015)
3. 3D Constrainted variational analysis (Tang and Zhang 2015)

Radar retrievals (North and Kollias 2015)

$$J = J_o + J_c + J_s + J_b + J_p.$$

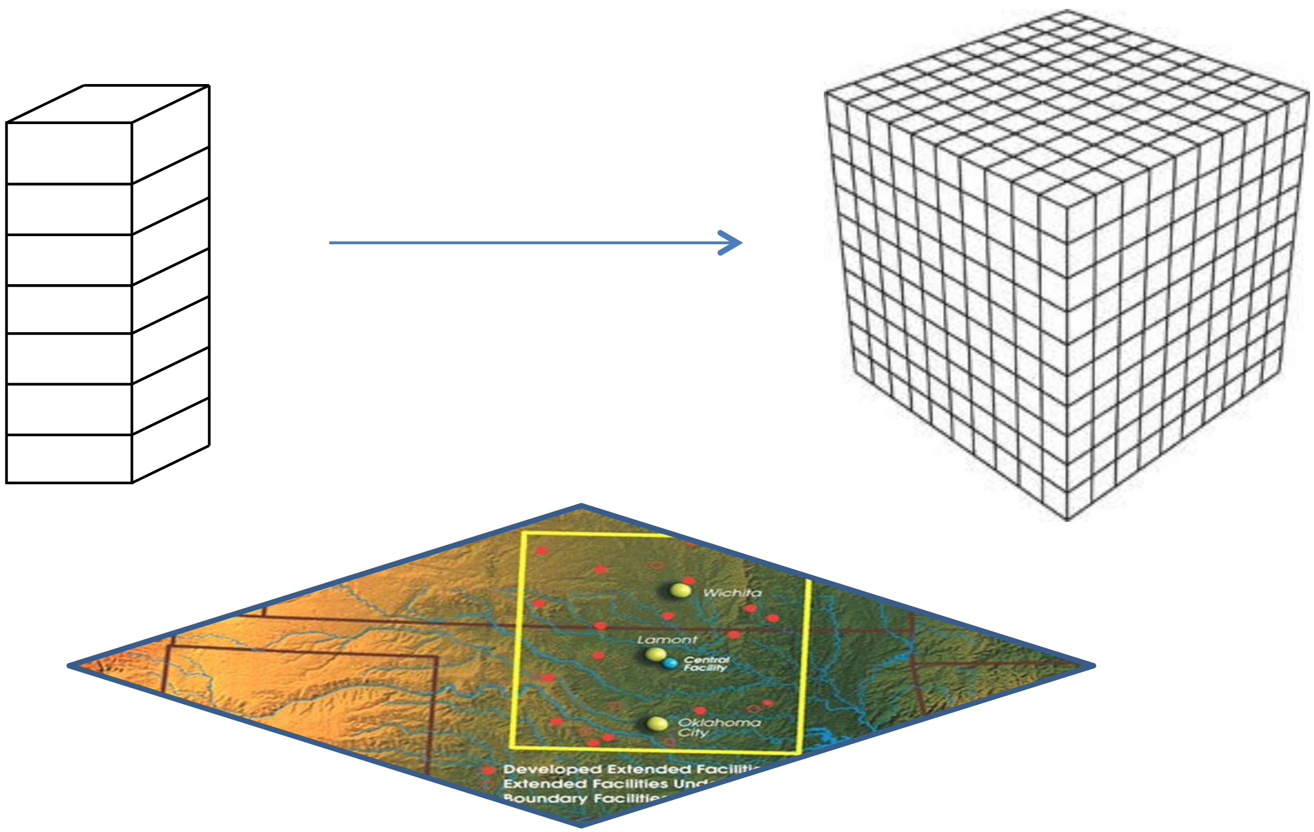
$$J_o = \frac{1}{2} \sum_{i,j,k} \lambda_o \left(V_r - V_{r,o}\right)^2 .$$

GSI multiscale (Li et al. 2015)

$$J_L\left(\delta x_L\right) = \frac{1}{2}\delta x_L^T B_L^{-1} \delta x_L + \frac{1}{2}\left(H\delta x_L - d\right)^T \left(R + HB_S H^T\right)^{-1} \left(H\delta x_L - d\right),$$

$$J_S\left(\delta x_S\right) = \frac{1}{2}\delta x_S^T B_S^{-1} \delta x_S + \frac{1}{2}\left(H\delta x_S - d\right)^T \left(R + HB_L H^T\right)^{-1} \left(H\delta x_S - d\right),$$

3D constrained variational analysis at SGP for a horizontal mesh of 9x10 grids at 0.5 degree resolution (Tang and Zhang 2016)



$$I(t) = (u - u_o)^{\mathrm{T}} \mathbf{B}_u^{-1} (u - u_o) + (v - v_o)^{\mathrm{T}} \mathbf{B}_v^{-1} (v - v_o) + (s - s_o)^{\mathrm{T}} \mathbf{B}_s^{-1} (s - s_o) + (q - q_o)^{\mathrm{T}} \mathbf{B}_q^{-1} (q - q_o), \tag{24-10}$$

$$A_{p_s}(V_{i,k}^*) = \left(\frac{\partial p_s}{\partial t}\right)_m + \sum_{k=1}^{K} (\nabla \cdot V_k^*)_m \Delta p_k = 0$$

$$A_q(V_{i,k}^*, q_{ik}^*) = \left\langle \left(\frac{\partial q^*}{\partial t}\right)_m \right\rangle_k + \left\langle \left(\nabla \cdot V^* q^*\right)_m \right\rangle_k - E_s + P_{rec} + \left\langle \left(\frac{\partial q_l}{\partial t}\right)_m \right\rangle_k = 0$$

$$A_s(V_{i,k}^*, s_{ik}^*) = \left\langle \left(\frac{\partial s^*}{\partial t}\right)_m \right\rangle_k + \left\langle \left(\nabla \cdot V^* s^*\right)_m \right\rangle_k - R_{TOA} + R_{srf} - LP_{rec} - SH + L \left\langle \left(\frac{\partial q_l}{\partial t}\right)_m \right\rangle_k = 0$$

$$A_V(V_{i,k}^*, \phi_{ik}^*) = \left\langle \left(\frac{\partial V^*}{\partial t}\right)_m \right\rangle_k + \left\langle \left(\nabla \cdot V^* V^*\right)_m \right\rangle_k + fk \times \left\langle \left(V^*\right)_m \right\rangle_k + \left\langle \left(\nabla \phi^*\right)_m \right\rangle_k - \tau_s = 0$$

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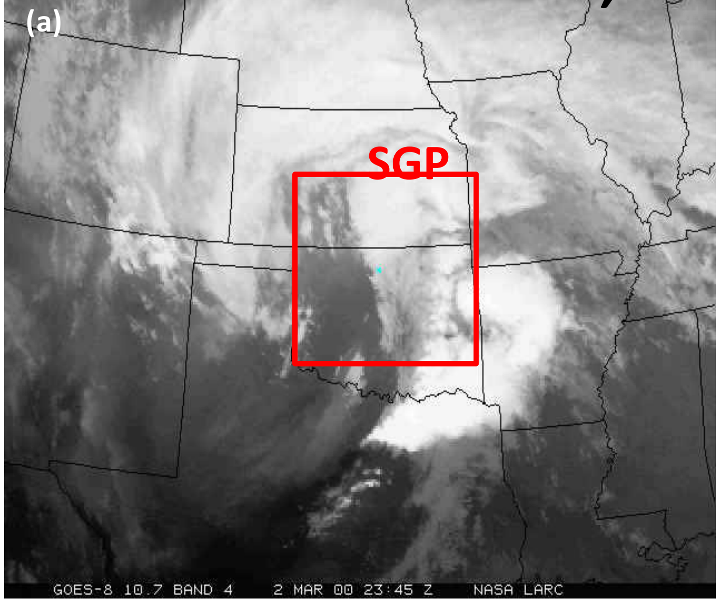
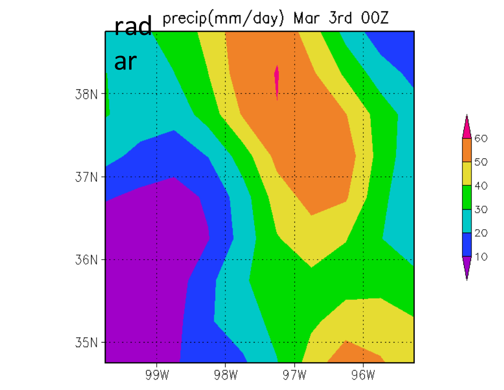
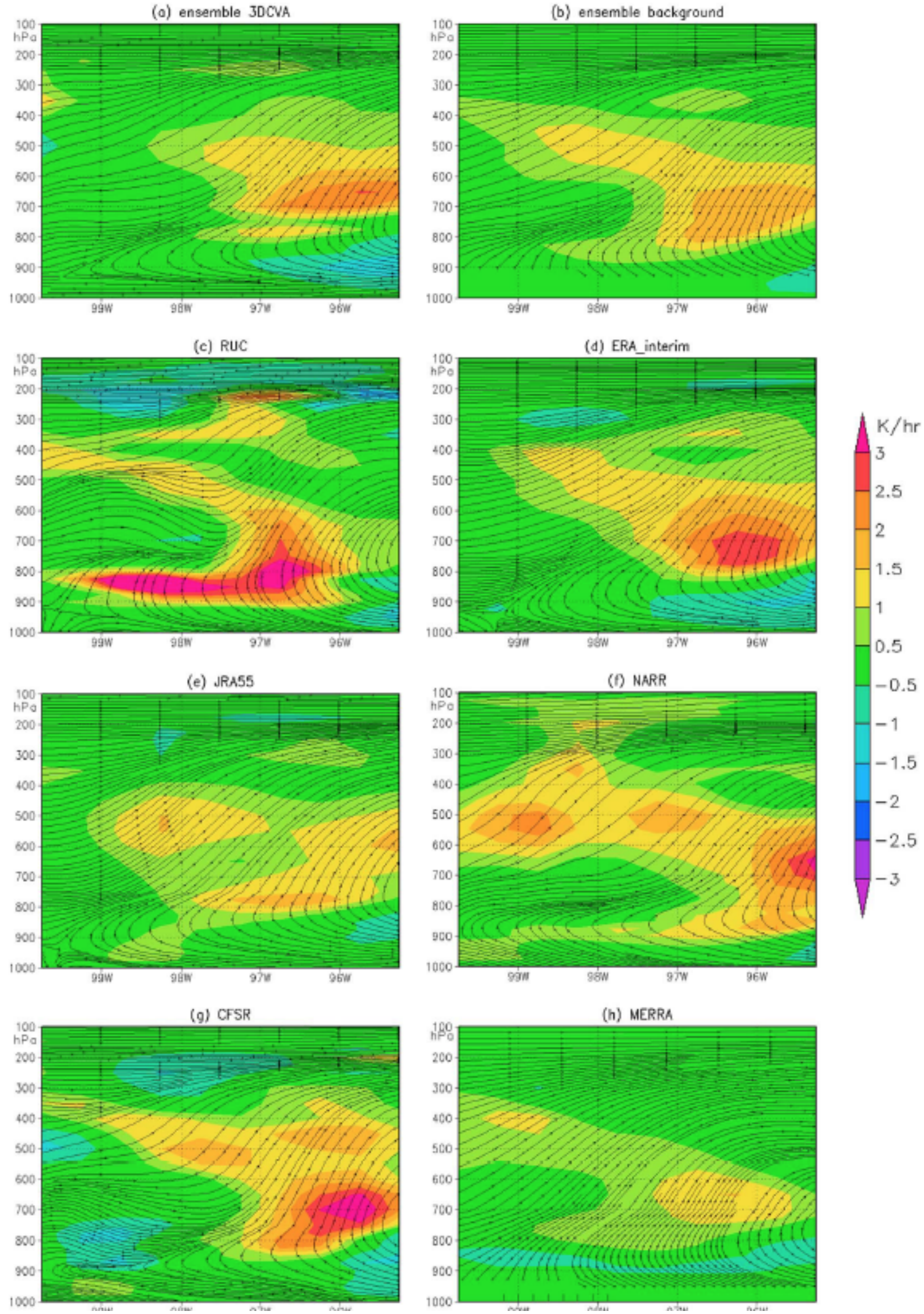
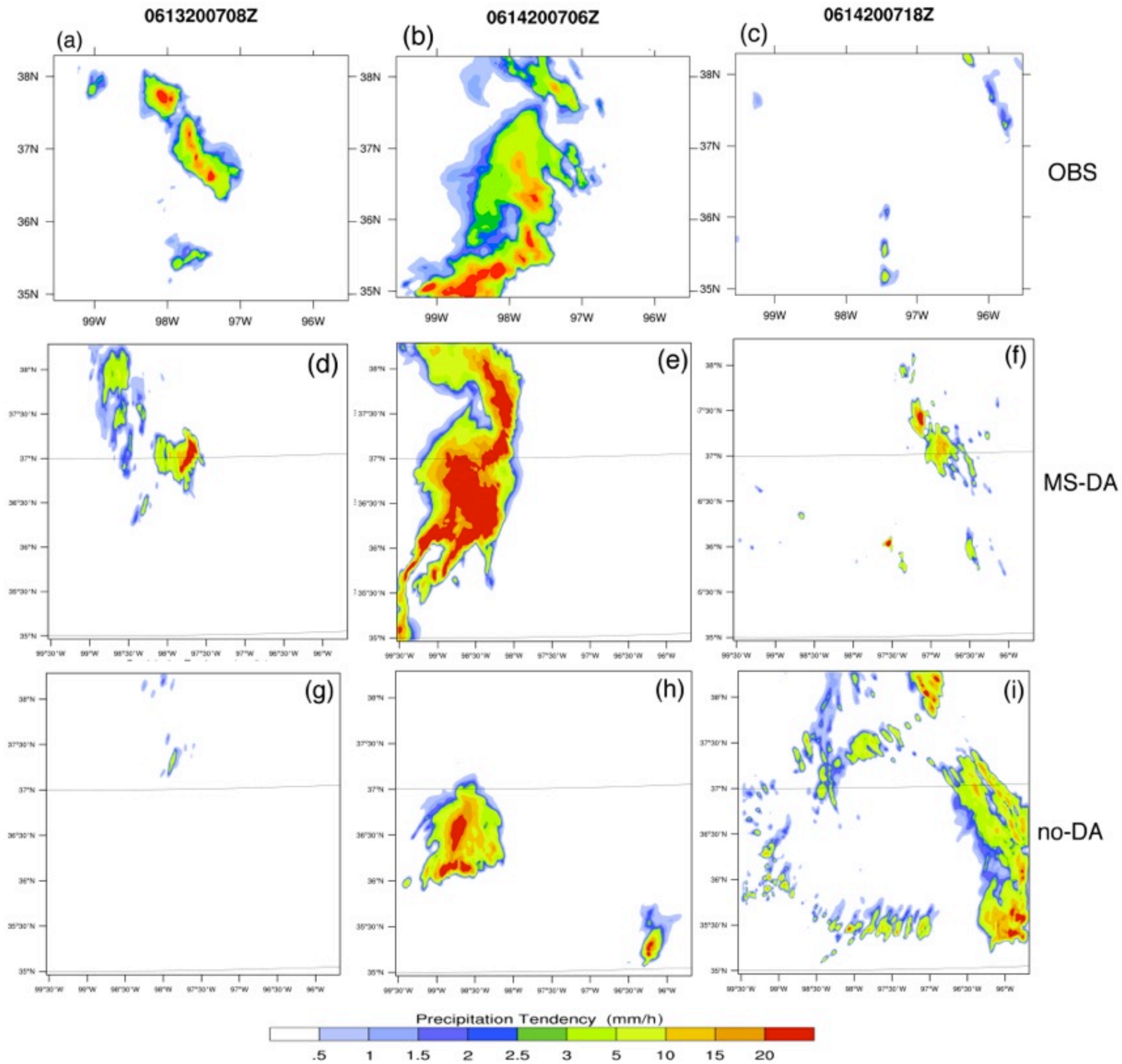






(from Tang and Zhang 2015)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20

The new method:

- To implement the variational constraints into the operational WRF GSI data assimilation system
- It can potentially incorporate some algorithms for used for radar retrievals

A deep dive into the GSI code.

To minimize

$$J(X) = X^T B^{-1} X - 2\ln\{W_t \exp(-0.5 * [H(X + Xb) - O)\hat{}2/R\hat{}2]) + W_g\}$$

- ✓ We formulated the GSI conjugate gradient method with constraints by using a new precondition matrix and algorithm

$$J(X,\lambda) = X^T B^{-1} X + \ln\left(w \exp(-[H(X+Xb) - O)\text{^}2/R\text{^}2] + u\right) + 2(AX - b)\lambda$$

$$dirx^{n+1} = -J_y^{n+1} + \beta \cdot dirx^n$$

$$diry^{n+1} = -J_x^{n+1} + \beta \cdot diry^n$$

$$\beta = -\frac{(J_x^{n+1} - J_x^n)^T \cdot J_y^{n+1}}{(J_x^{n+1} - J_x^n)^T \cdot J_y^n}$$

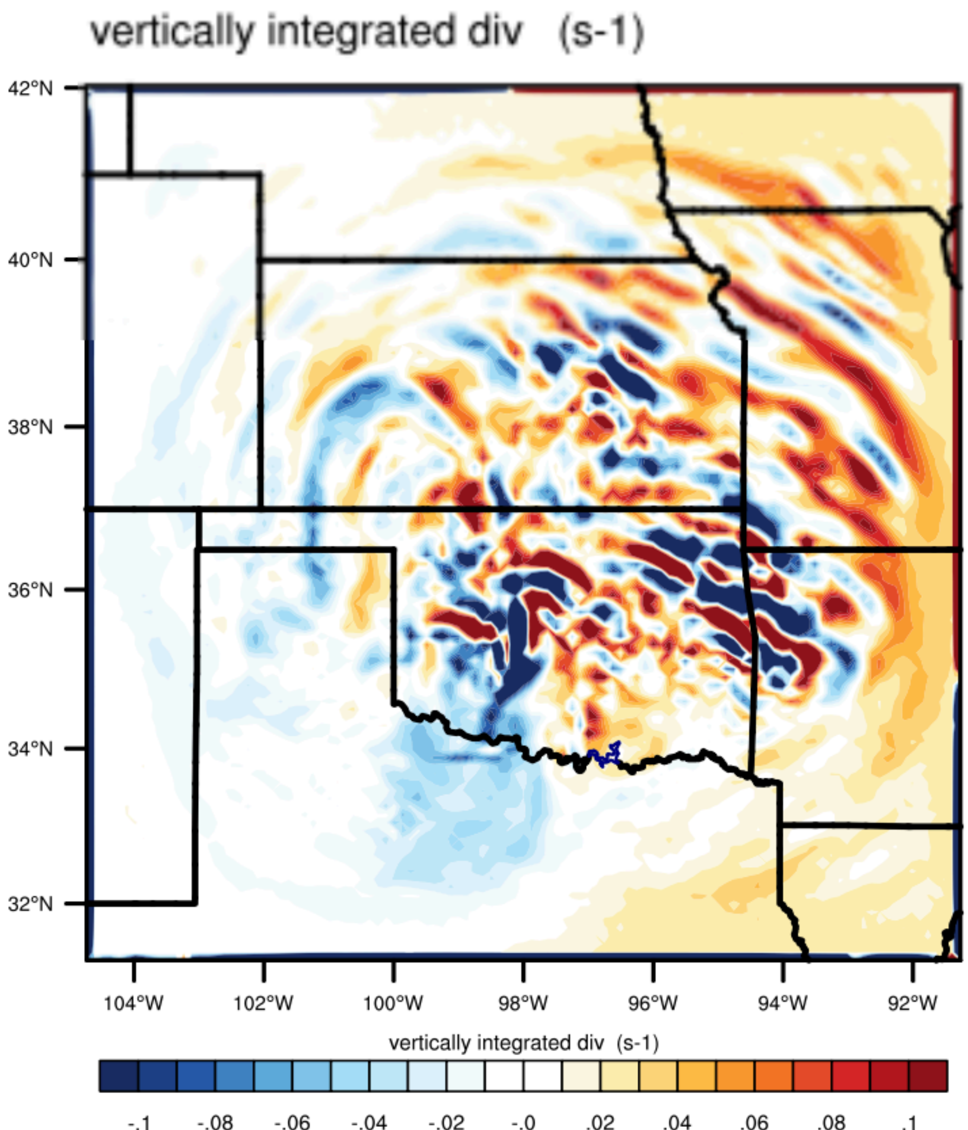
$$x^{n+1} = x^n + \alpha \cdot dirx^n$$

$$y^{n+1} = y^n + \alpha \cdot diry^n$$

$$\alpha = -\frac{(J_x^n)^T \cdot (J_y^n)}{(dirx^n)^T \cdot [\begin{pmatrix} I & A^T \\ AB & I \end{pmatrix} diry^n + \begin{pmatrix} H^T R^{-1} H \\ 0 \end{pmatrix} dirx^n]}$$

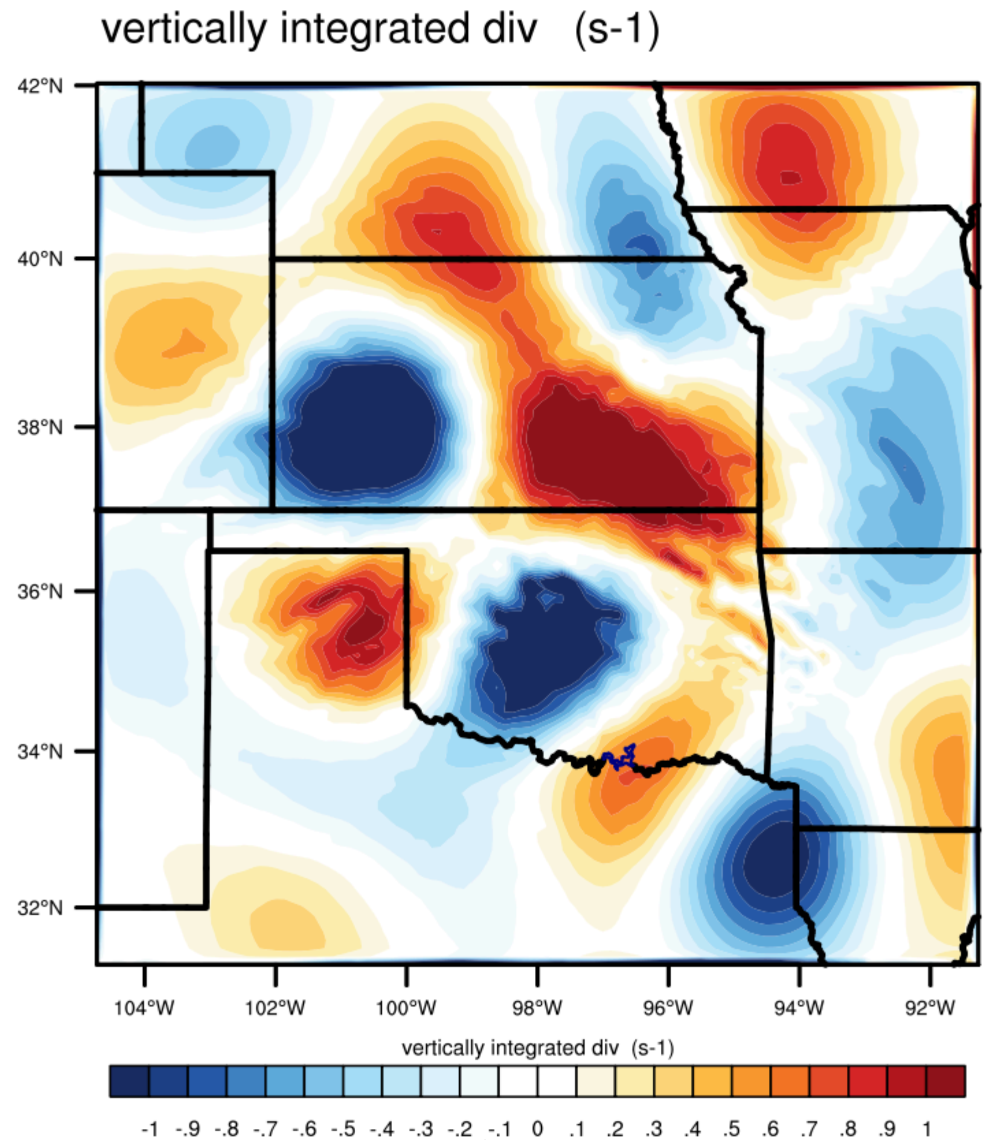
Mass budget:divergence

$$\left[\frac{\partial \mu_d}{\partial t}\right]_\eta + \int_0^1 \nabla_\eta \left(\vec{V}_h \mu_d\right) \mathrm{d}\eta = 0$$



Before GSI

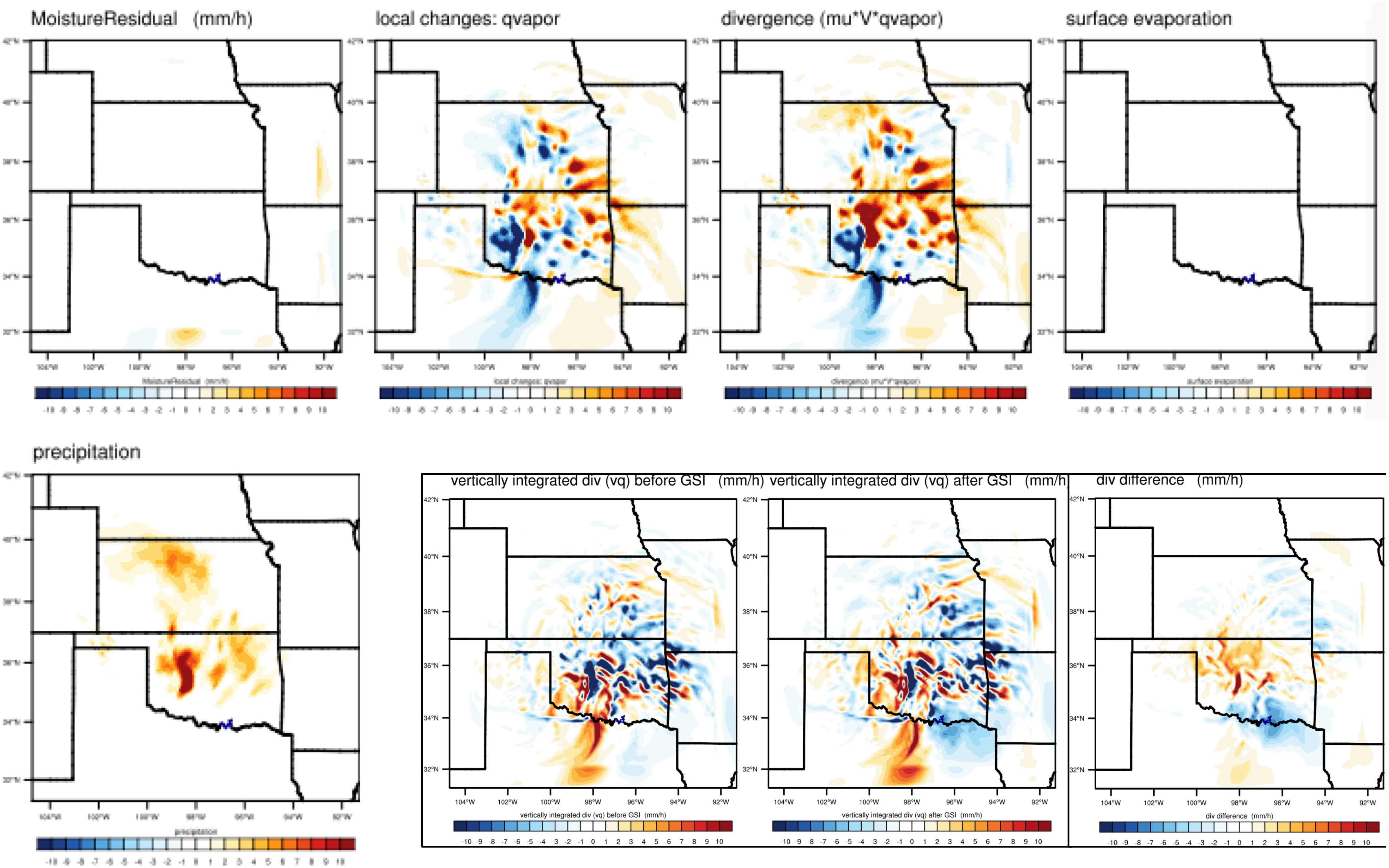
Contour: [-0.1, 0.1]; units: Pa s-1



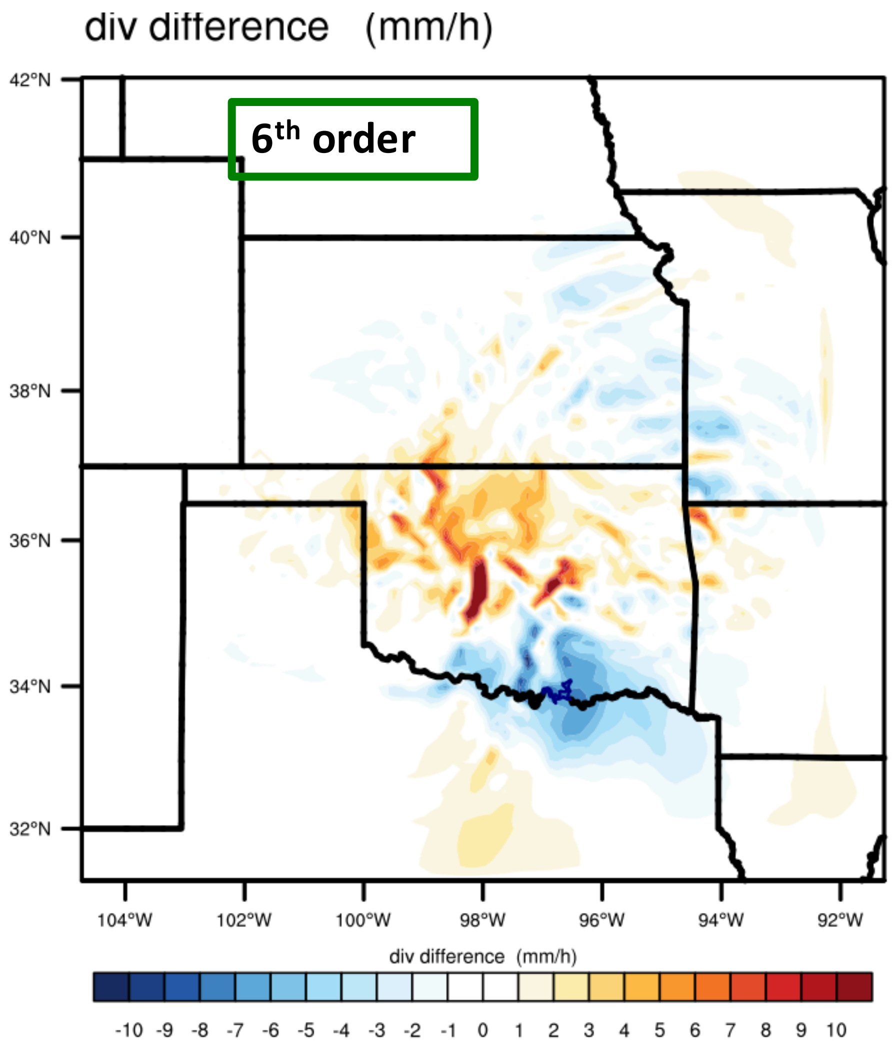
After GSI

Contour: [-1.0, 1.0] ; units: Pa s-1

Water budget $\frac{1}{g}\int_0^1 \frac{\partial \pi^* q}{\partial t} \mathrm{d}\eta + \frac{1}{g}\int_0^1 \nabla\left(\pi^* q \vec{V}\right) \mathrm{d}\eta = (SFCEVP - RAIN)$



Moisture budget: divergence term

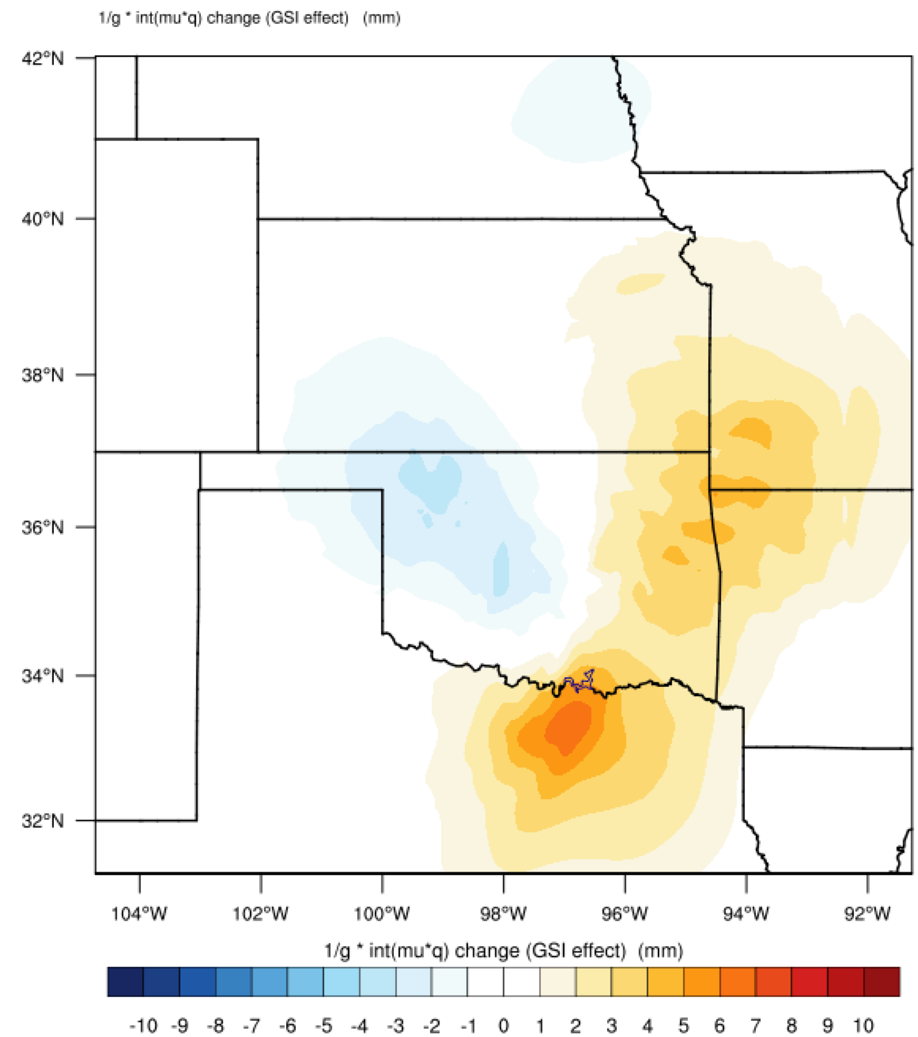


After GSI – before GSI

Contour: [-10.0, 10.0] mm/h

$$\frac{1}{g}\int_0^1 \nabla\left(\pi^* q\vec{V}\right)\mathrm{d}\eta$$

q: water vapor



After GSI – before GSI

Contour: [-10.0, 10.0] mm

$$\frac{1}{g}\int_0^1 \frac{\partial \pi^* q}{\partial t}\mathrm{d}\eta$$

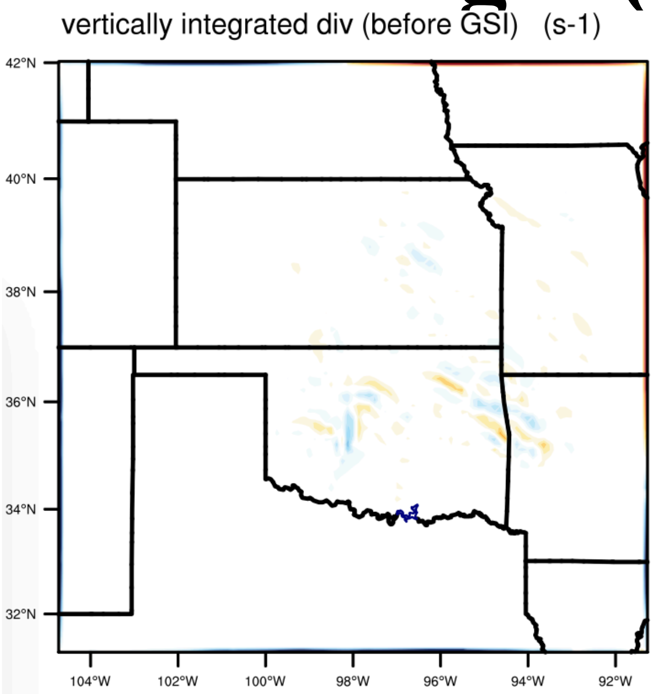
The modified assimilation algorithm will conserve mass and total water

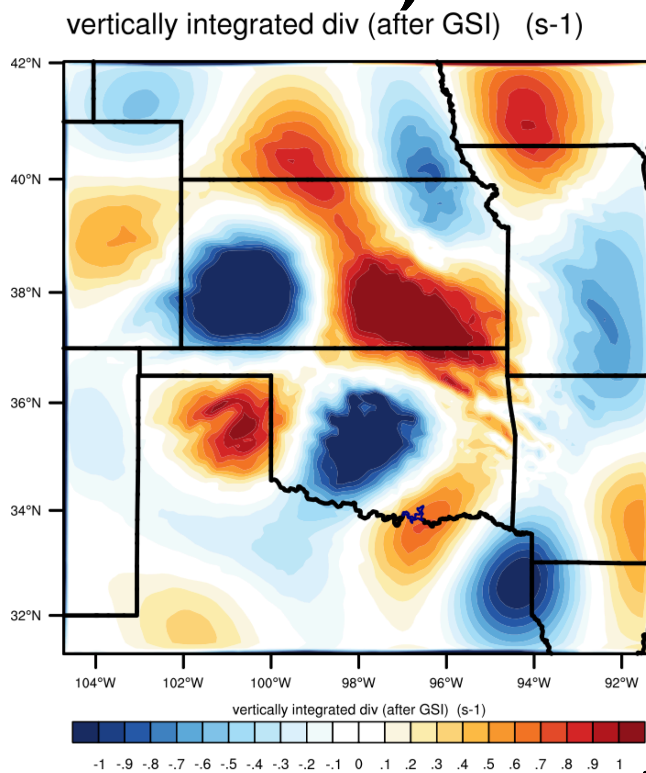
Work to implement the constraints in GSI is in progress. Results will be reported later.

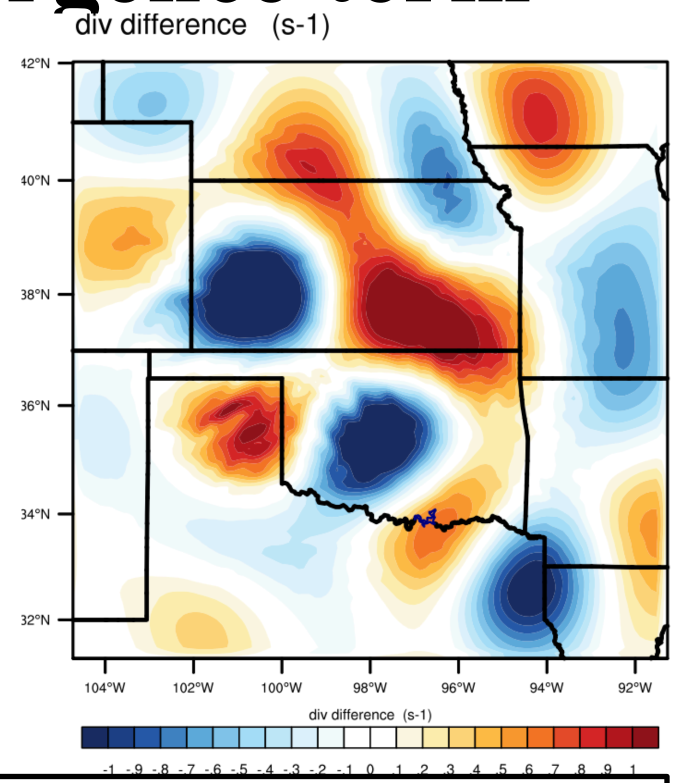
Comments Welcome!

Thank you!

Mass budget (Mercator): divergence term

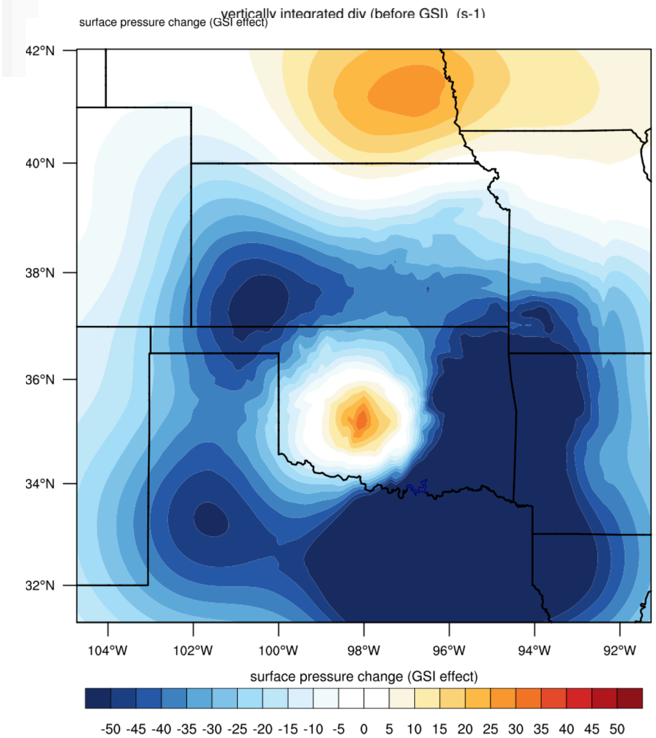






Divergence change due to GSI
Contour: [-1.0, 1.0]; units: Pa s-1

$$\left[\frac{\partial \mu_d}{\partial t}\right]_\eta + \int_0^1 \nabla_\eta \left(\vec{V}_h \mu_d\right) d\eta = 0$$



mu change due to GSI
Contour: [-50.0, 50.0]; units: Pa

$$\left[\frac{\partial \mu_d}{\partial t}\right]_\eta + \int_0^1 \nabla_\eta \left(\vec{V}_h \mu_d\right) d\eta = 0$$